RNN's and BMI's: linking network dynamics to behavior

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CoMPLEX
University College London

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March 5th, 2020

learning

How do brains learn?

motor learning

How do brains learn to produce goal-directed movements?

motor learning

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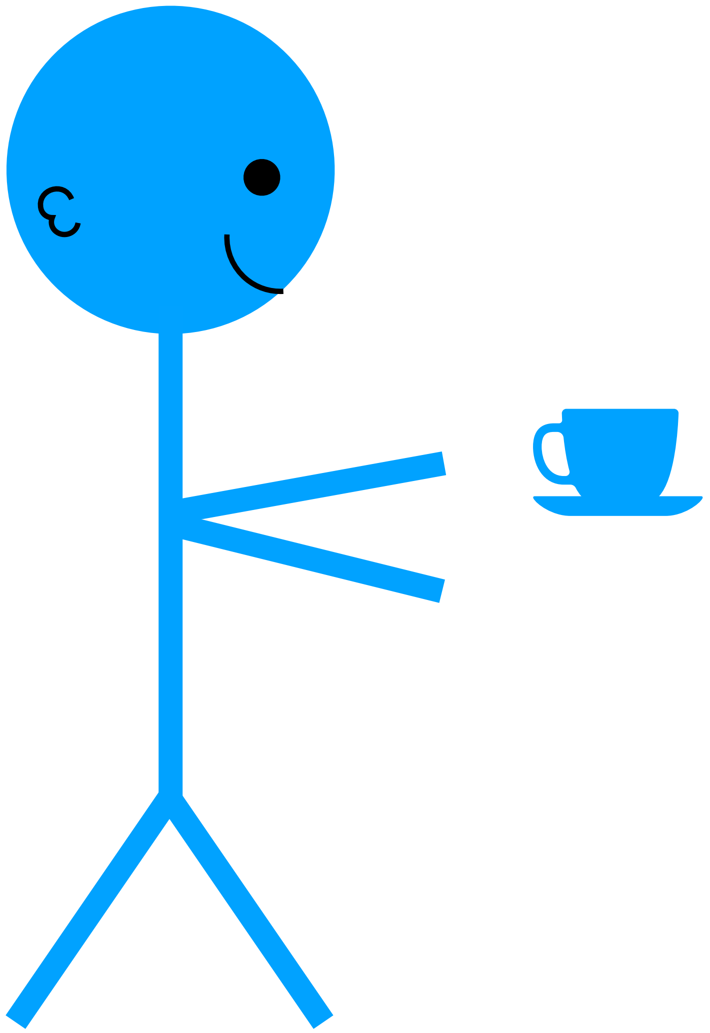

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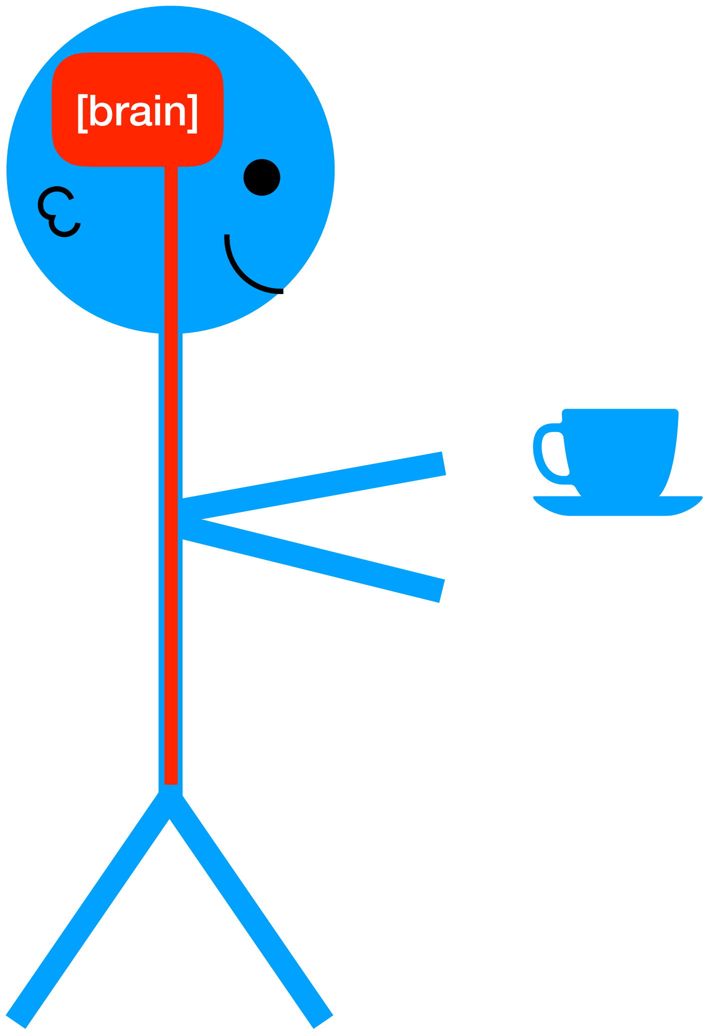

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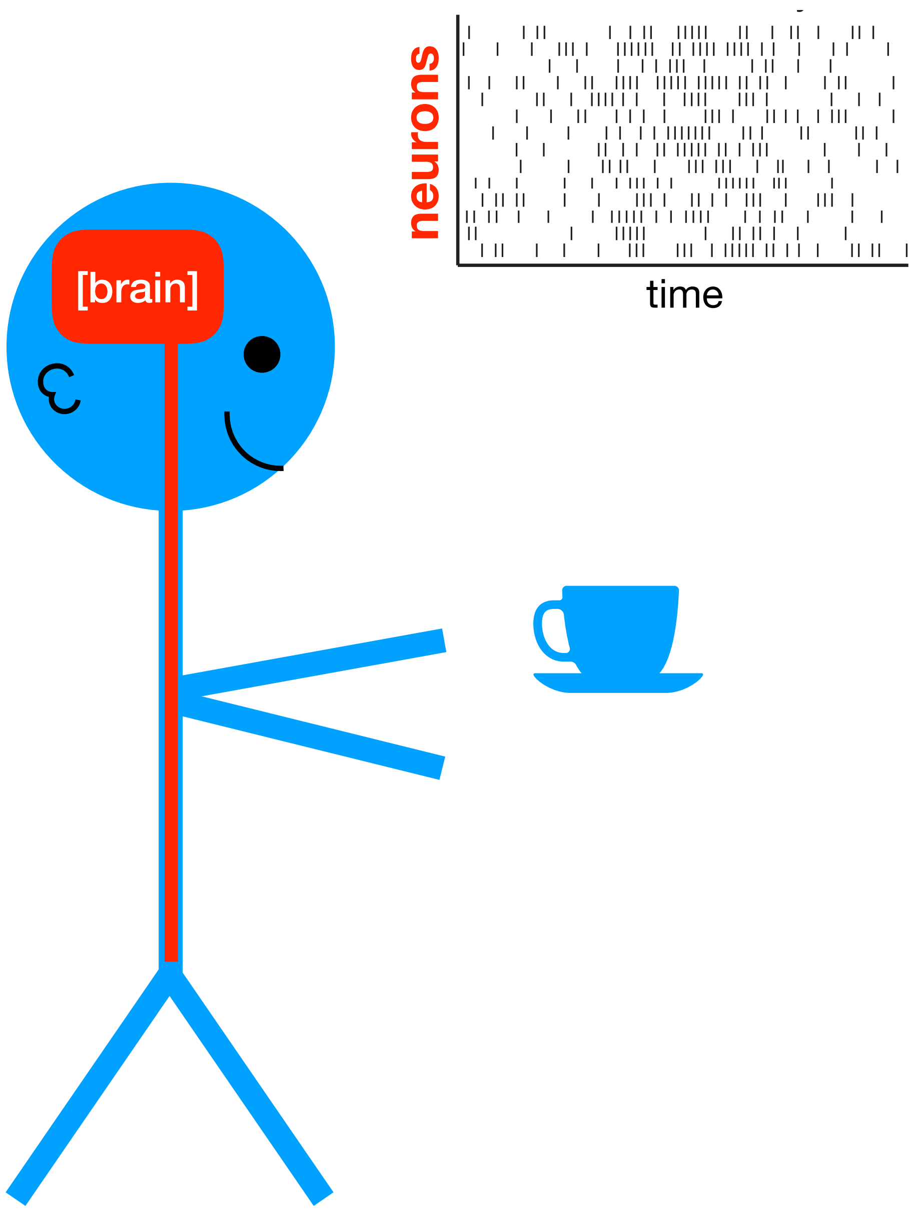

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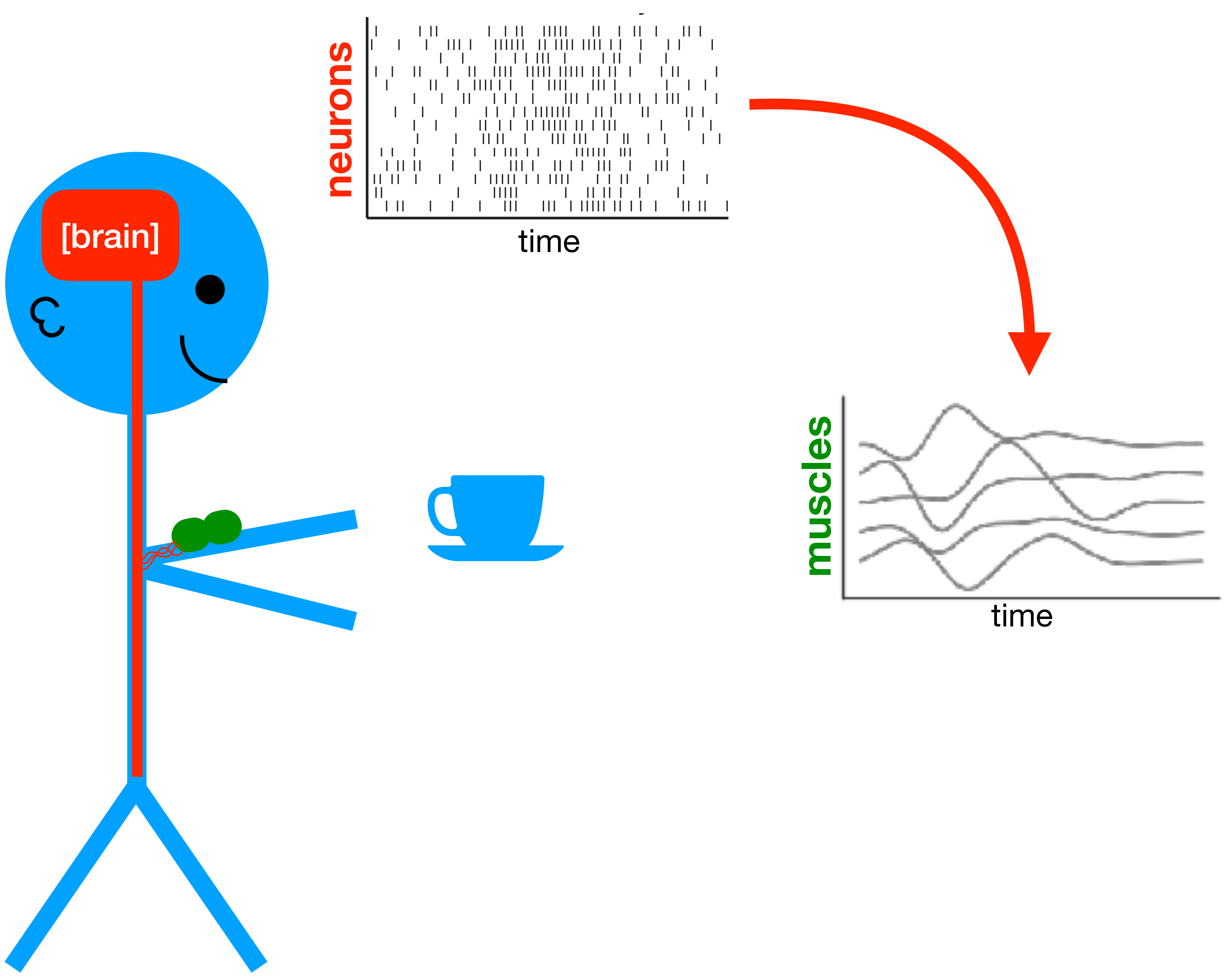

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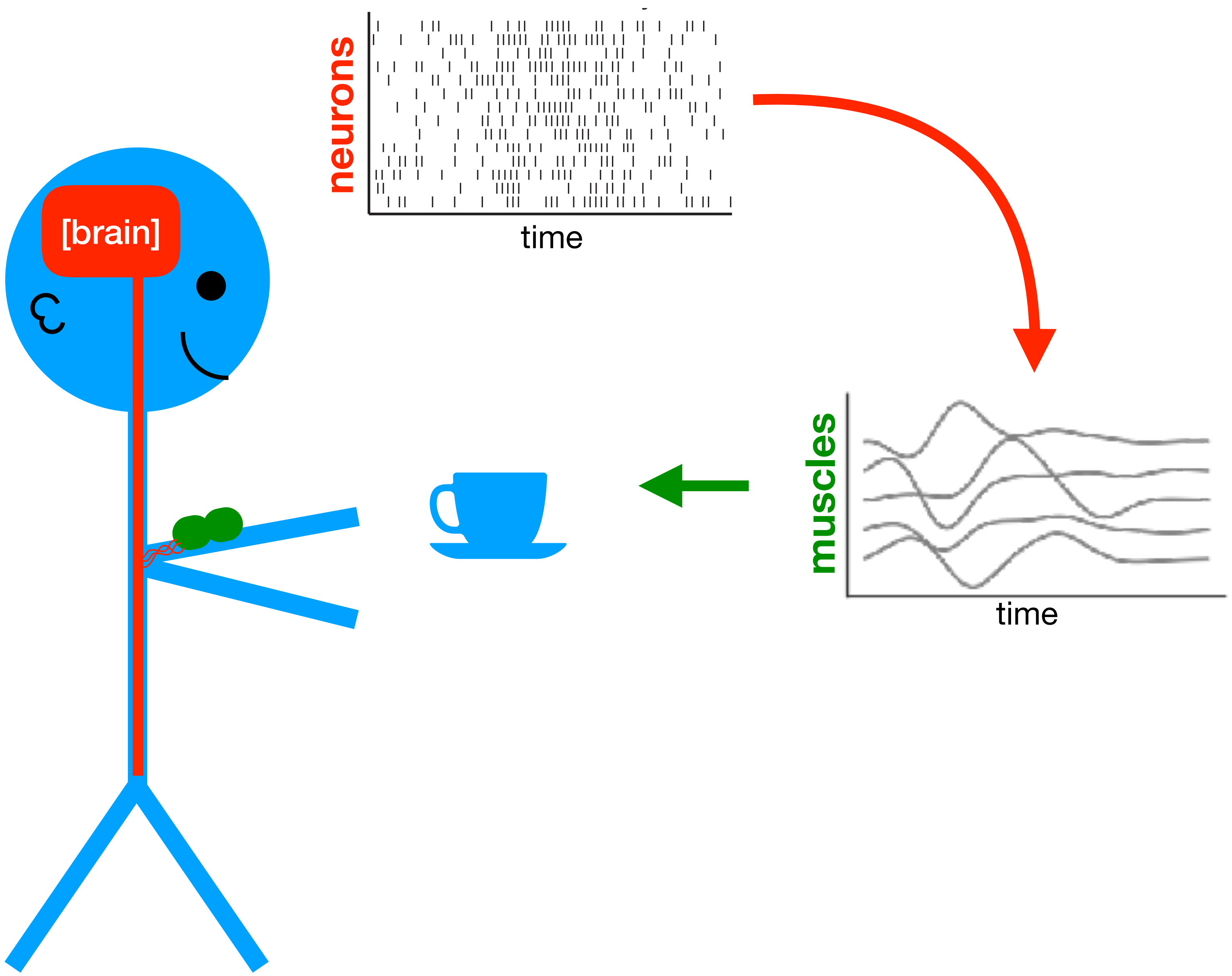
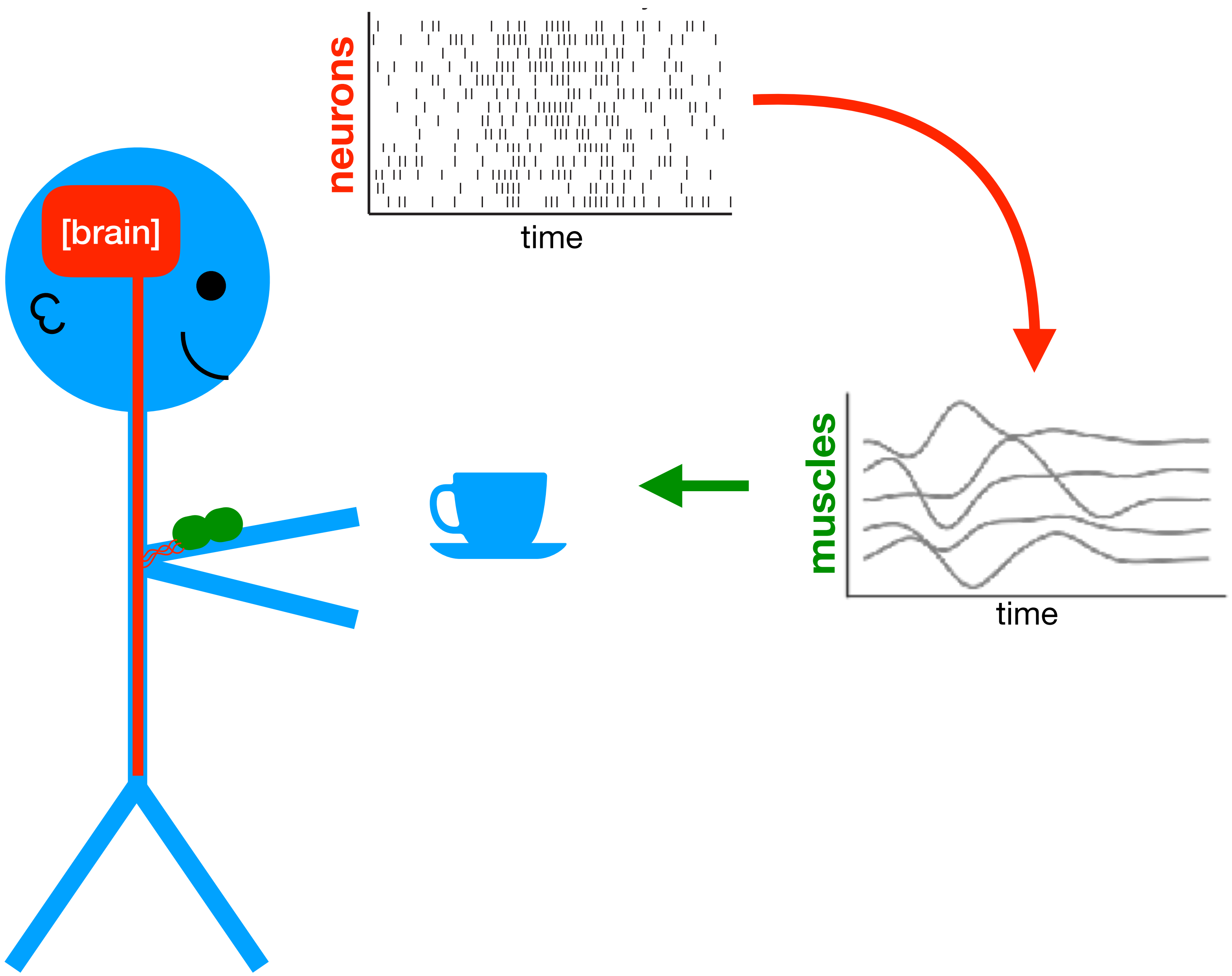


motor learning

How do brains learn to produce **spatiotemporal patterns of activity**?



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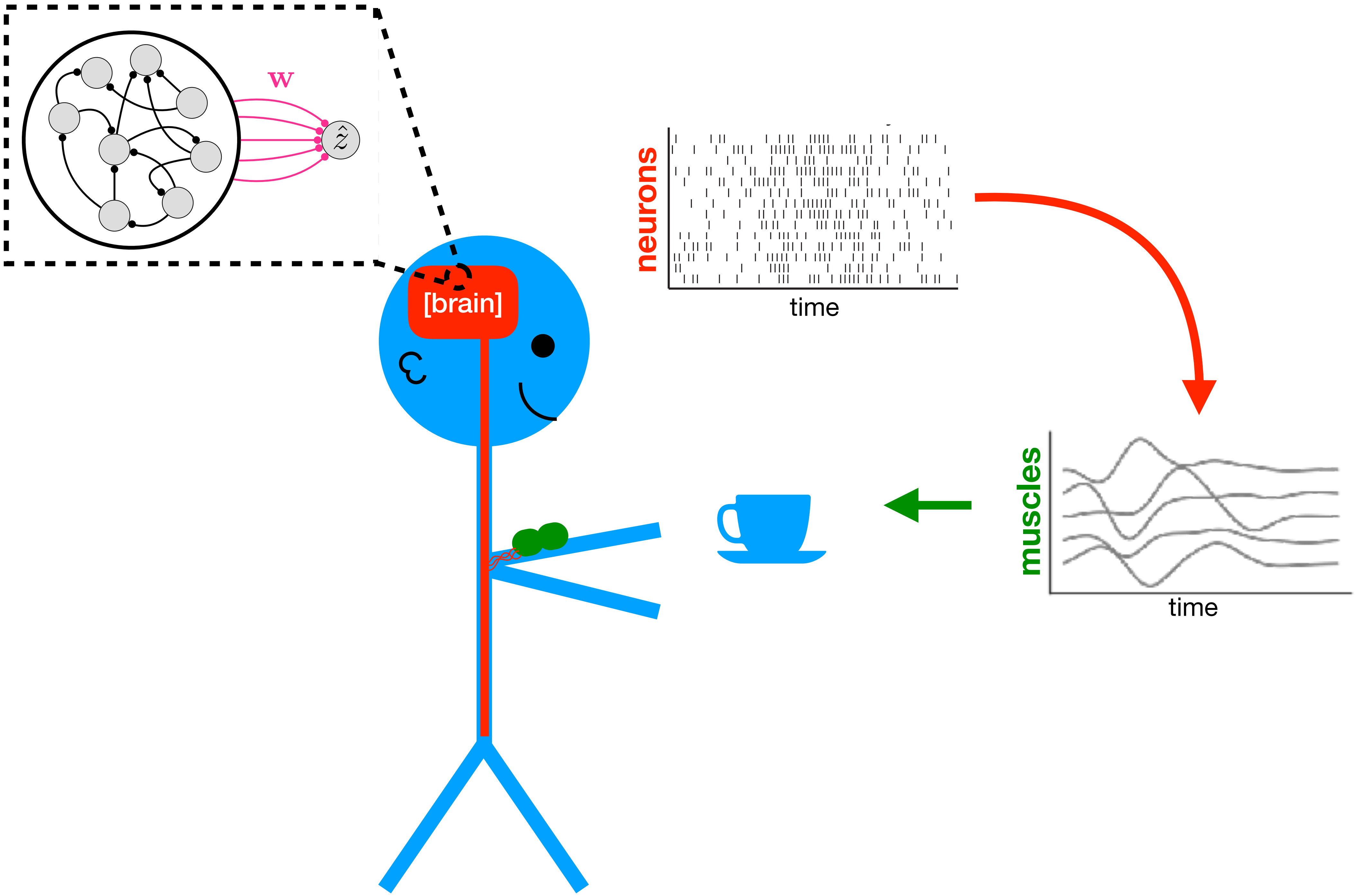

[**Menendez** & Latham (*AREADNE*, 2018)]

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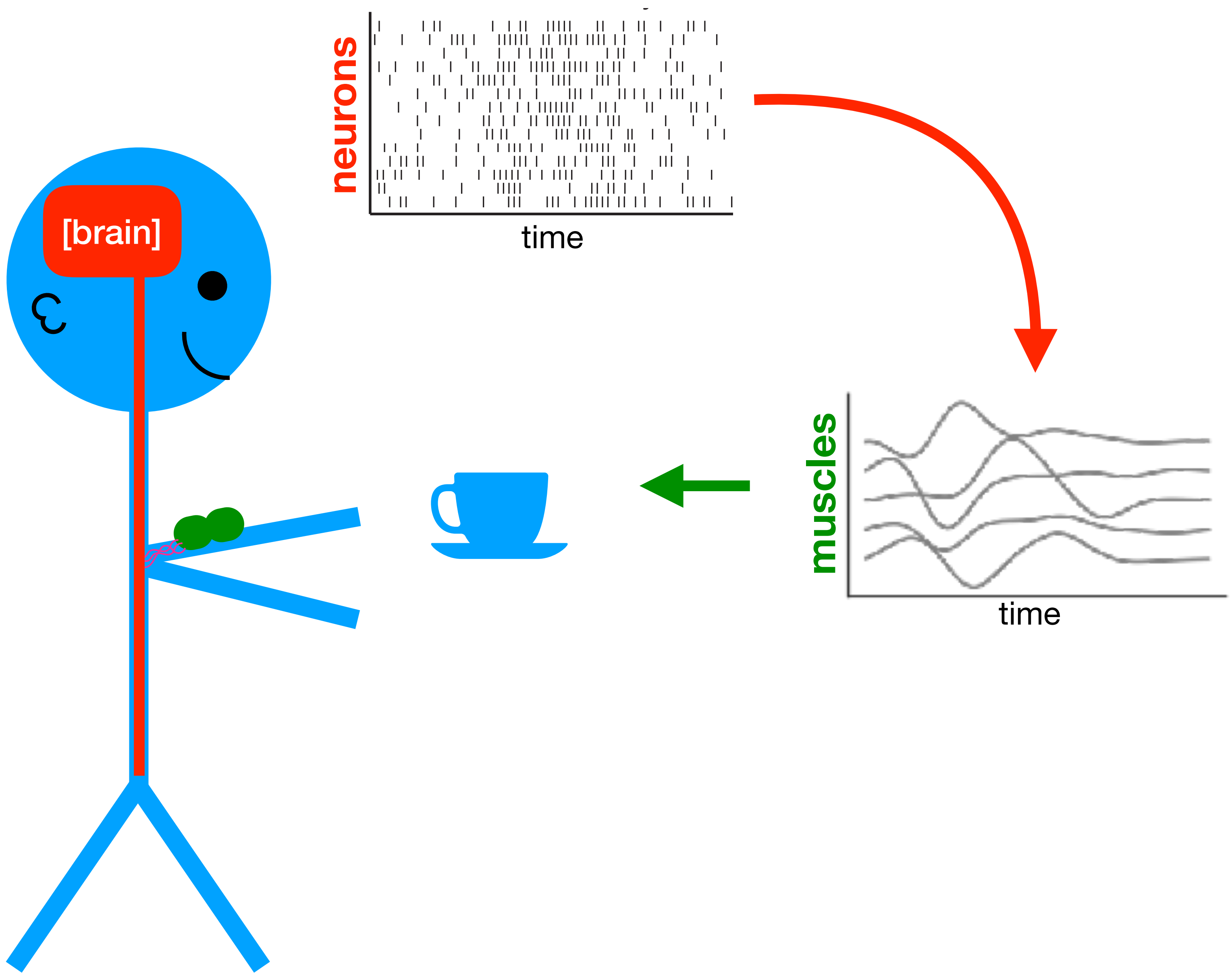

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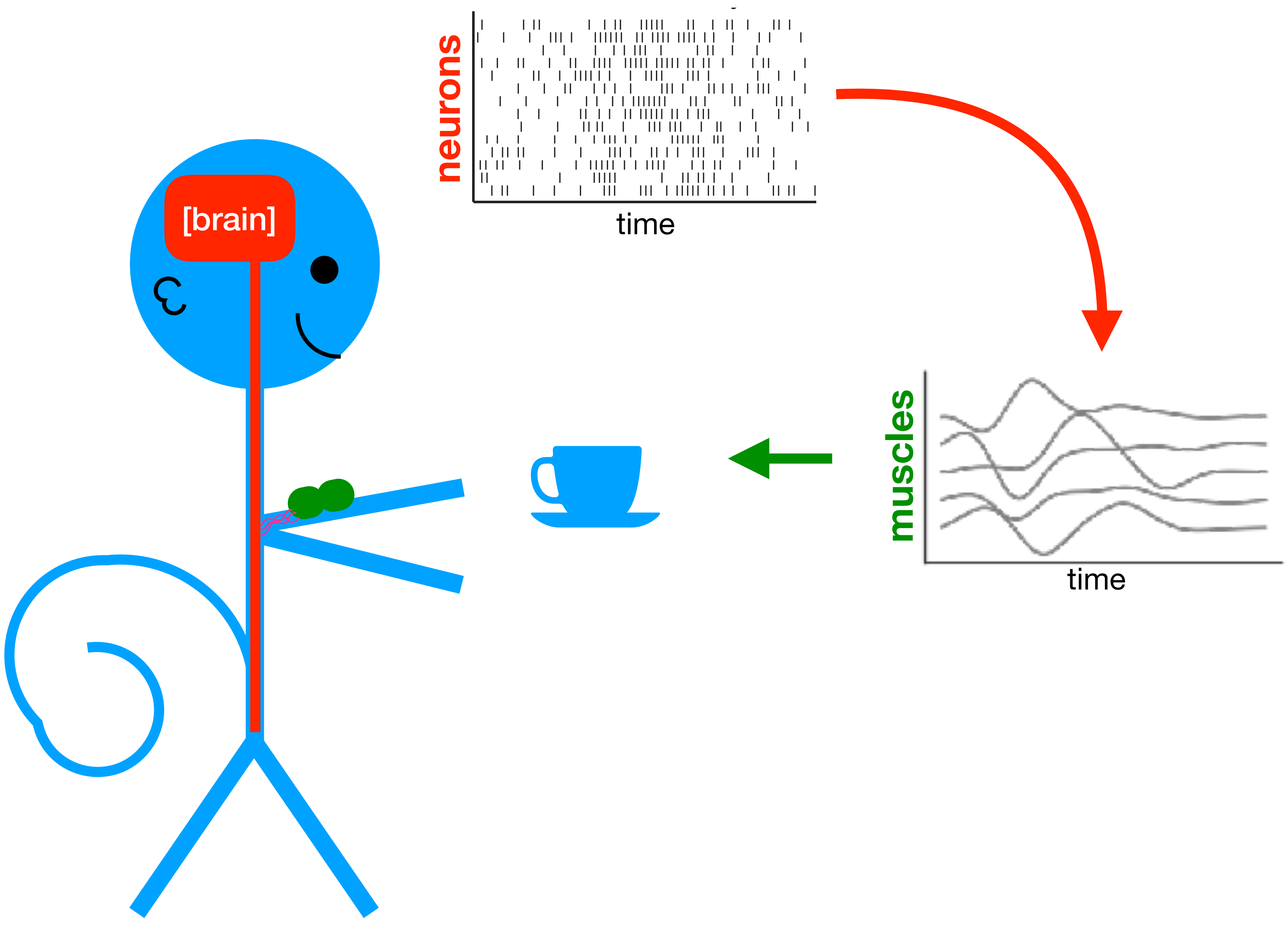

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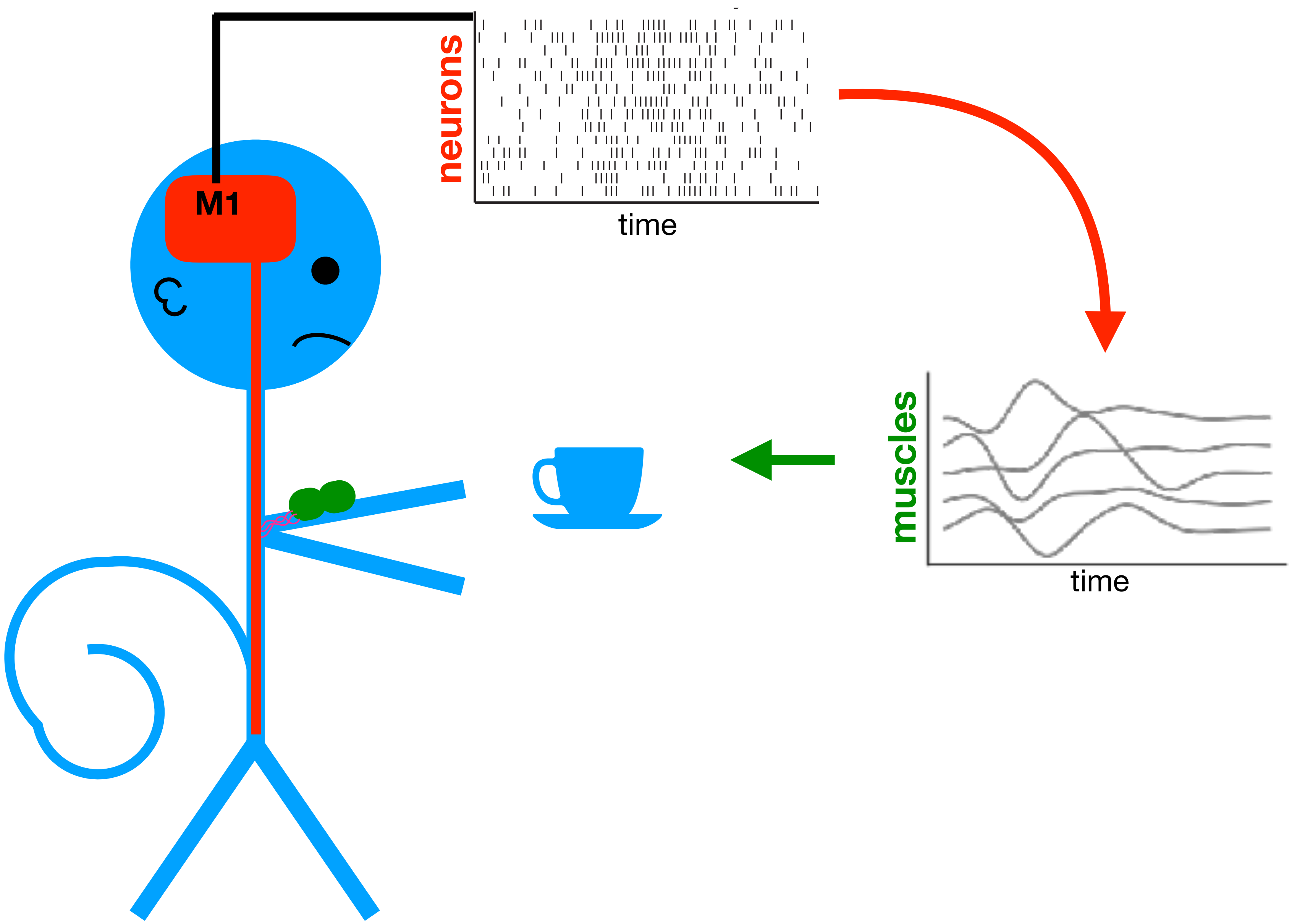
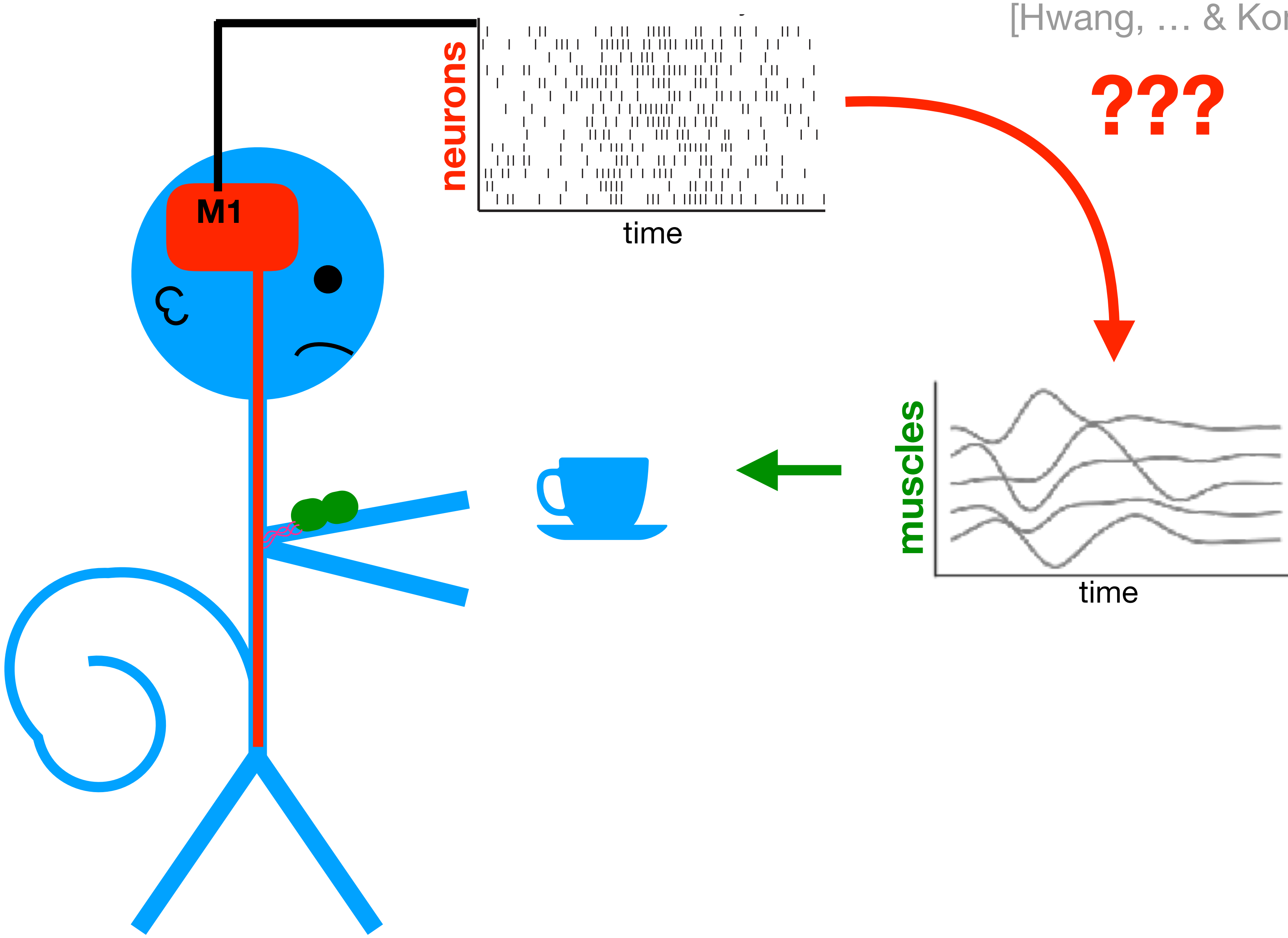


motor learning

How do brains learn to produce **spatiotemporal patterns of activity**?

[Kawai, … & Olveczky '15]
[Lopes, …, **Menendez**, … & Kampff '17]
[Hwang, … & Komiyama '19]



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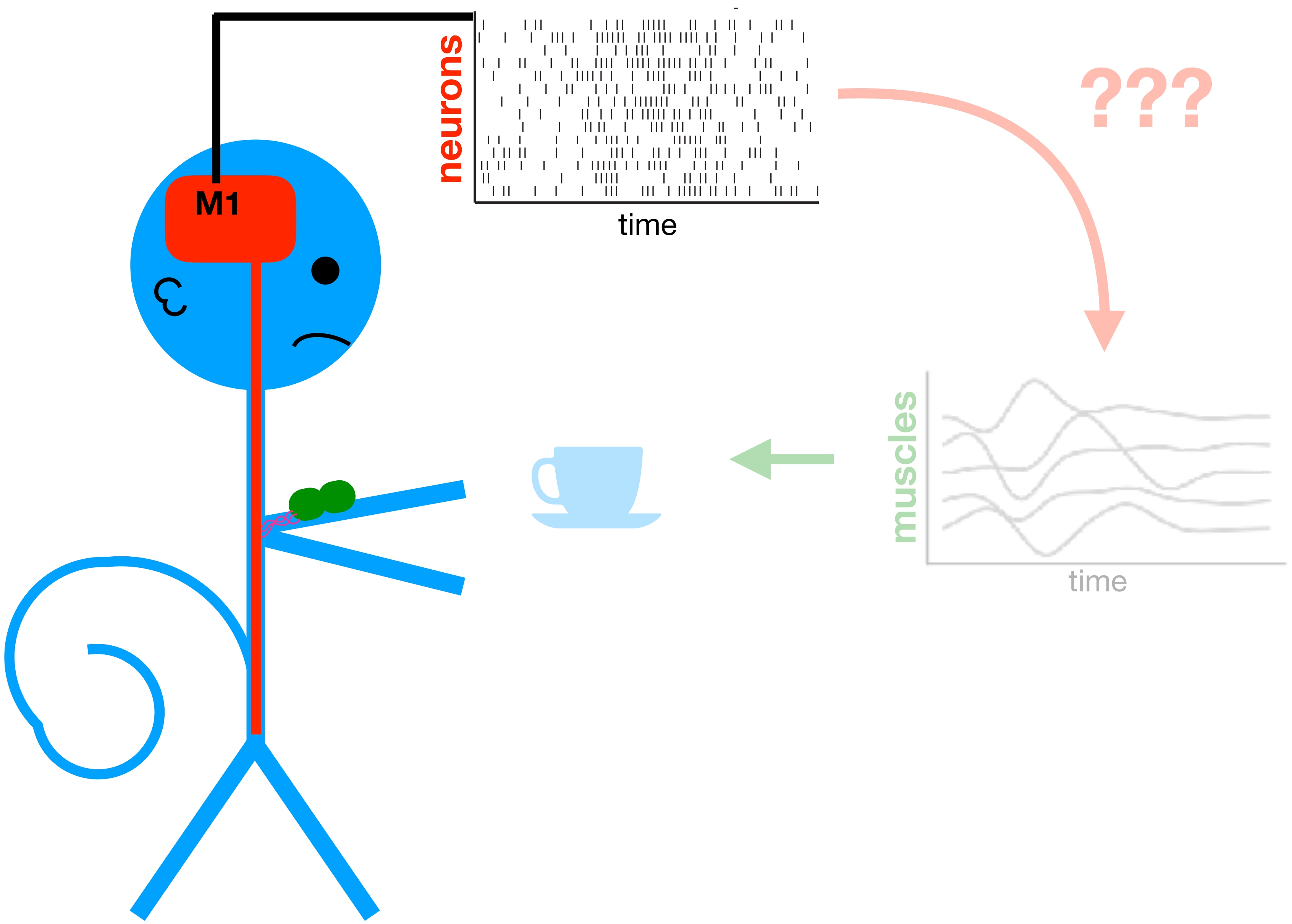

BMI learning

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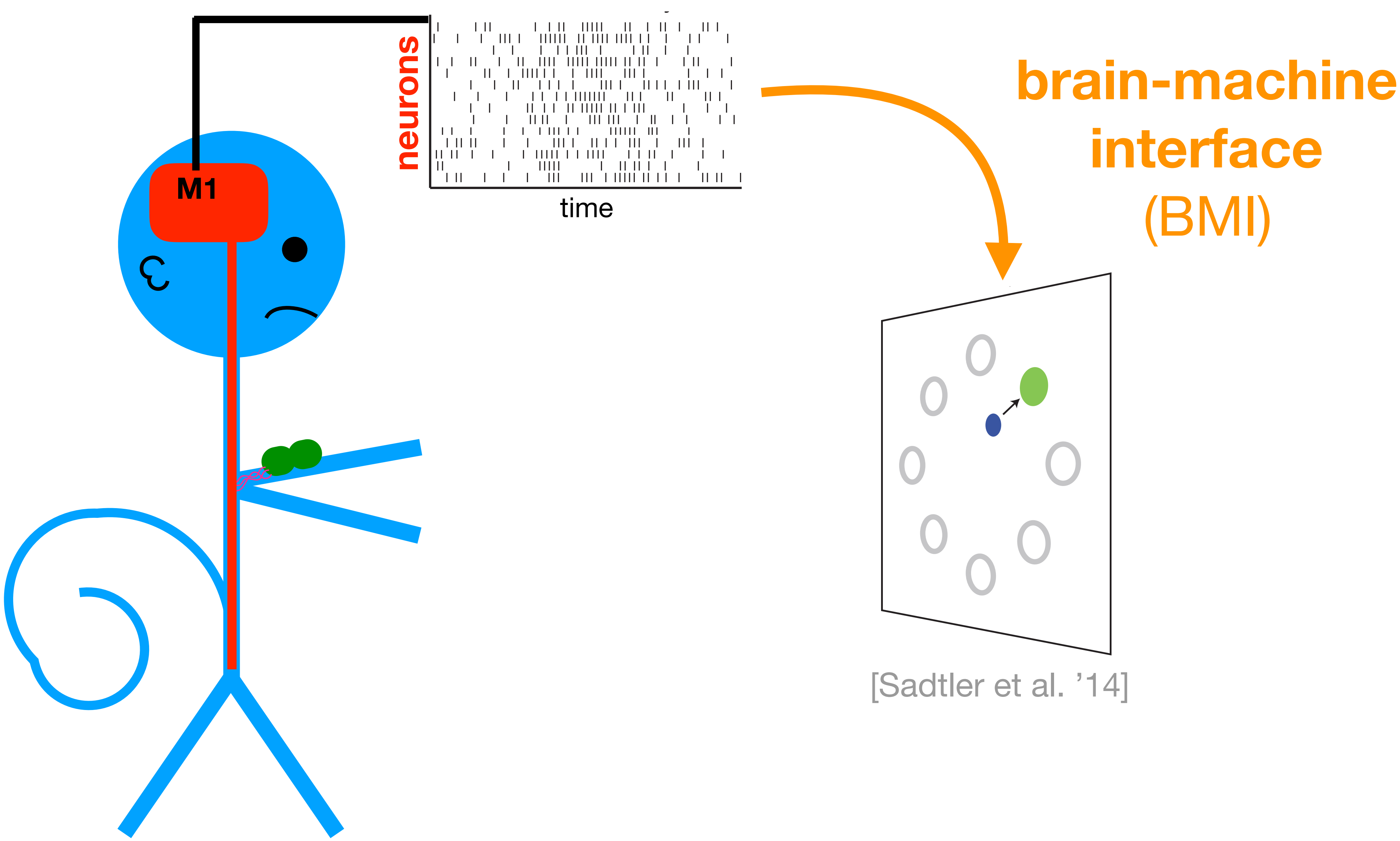

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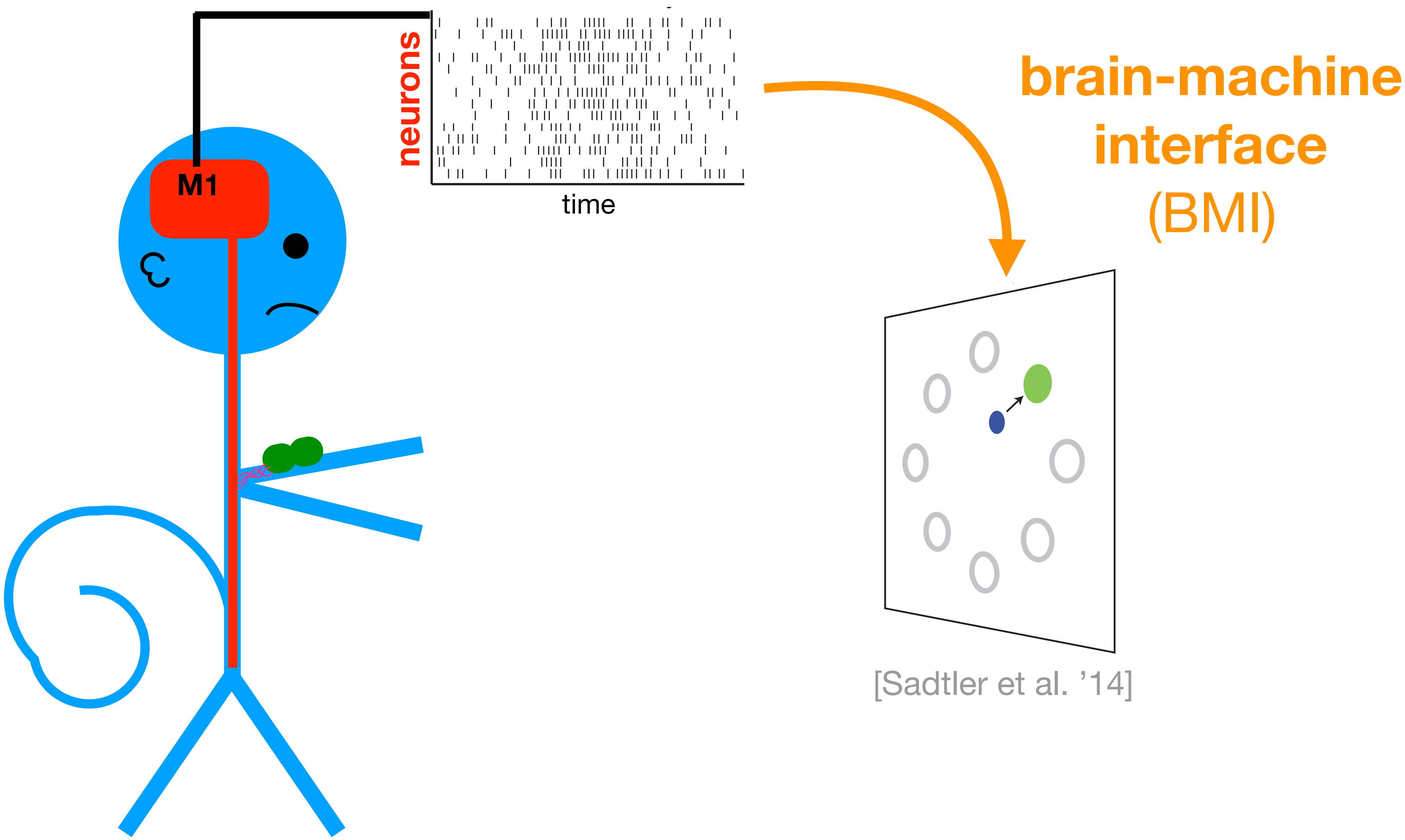
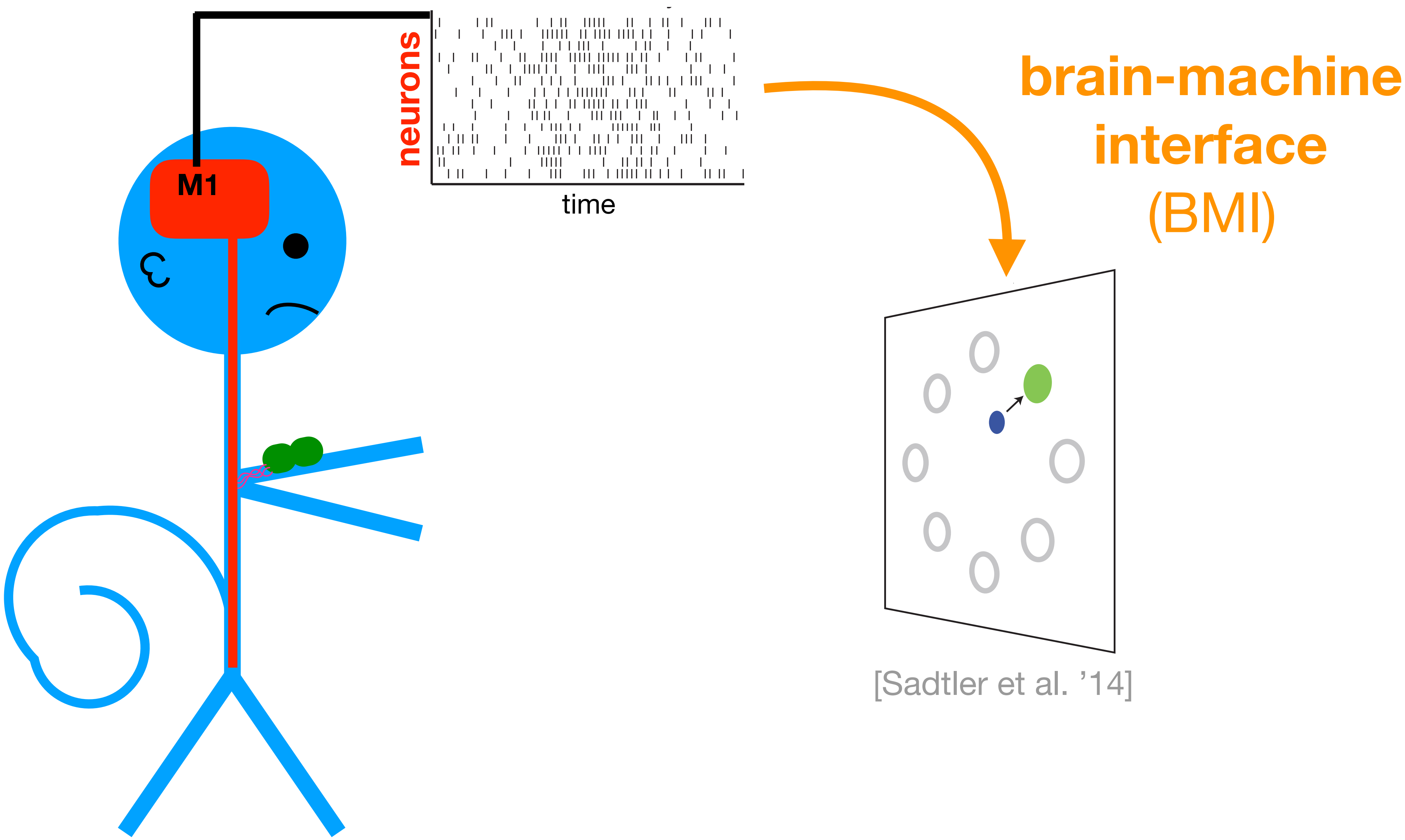


[Sadtler et al. '14]

BMI learning

Which neurons are relevant to this task?

[Legenstein et al. '10]



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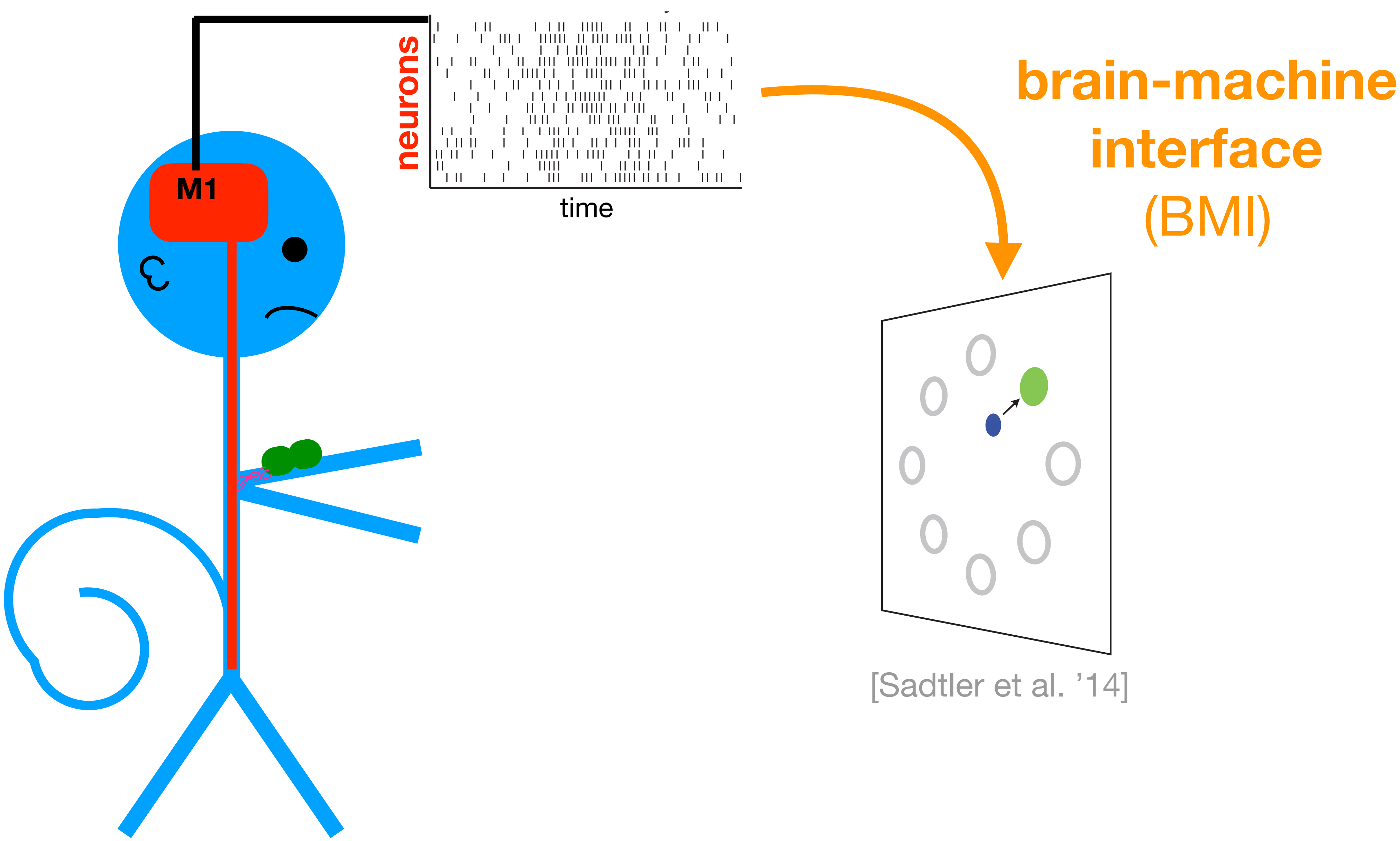

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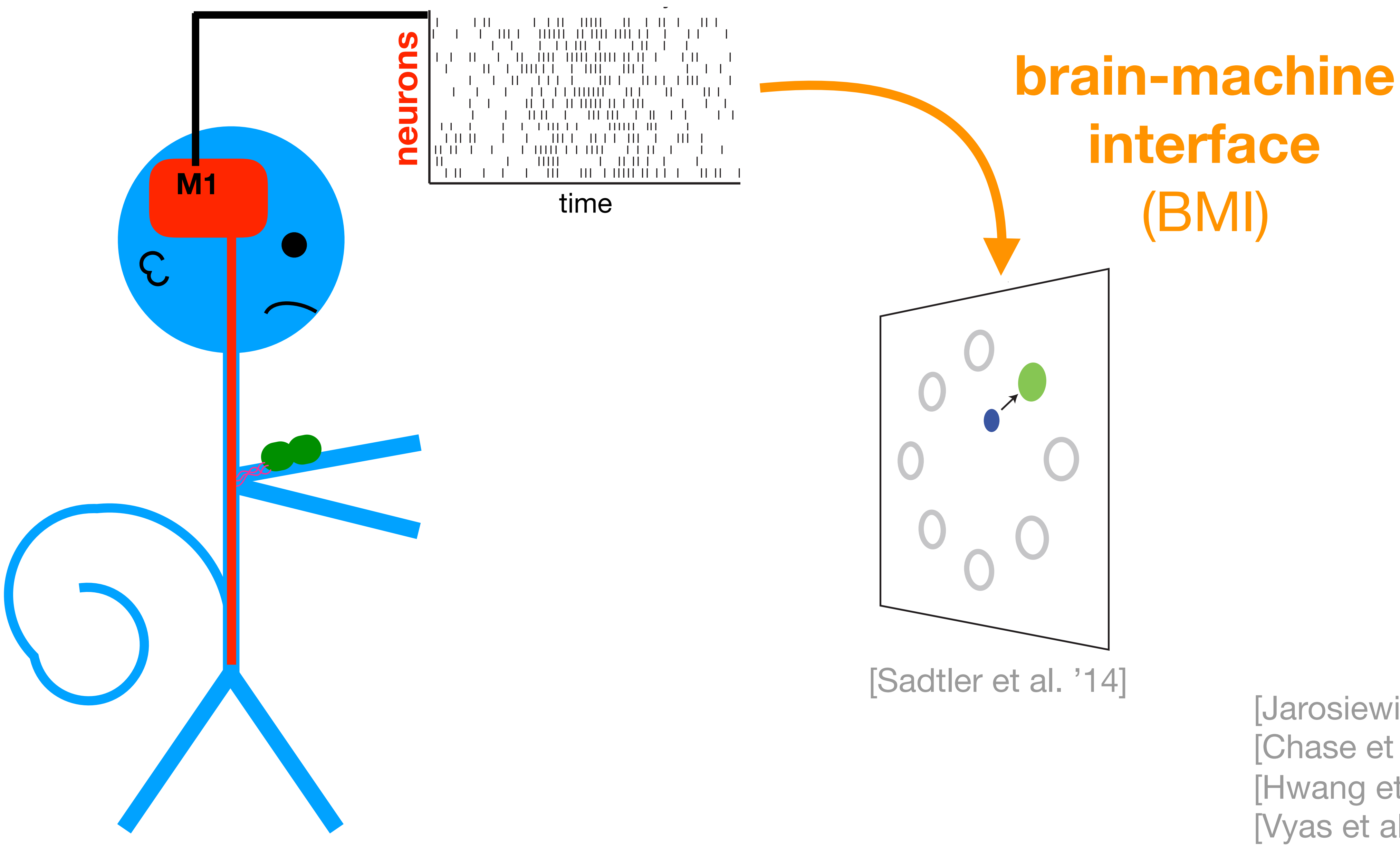
BMI learning

"re-aiming"



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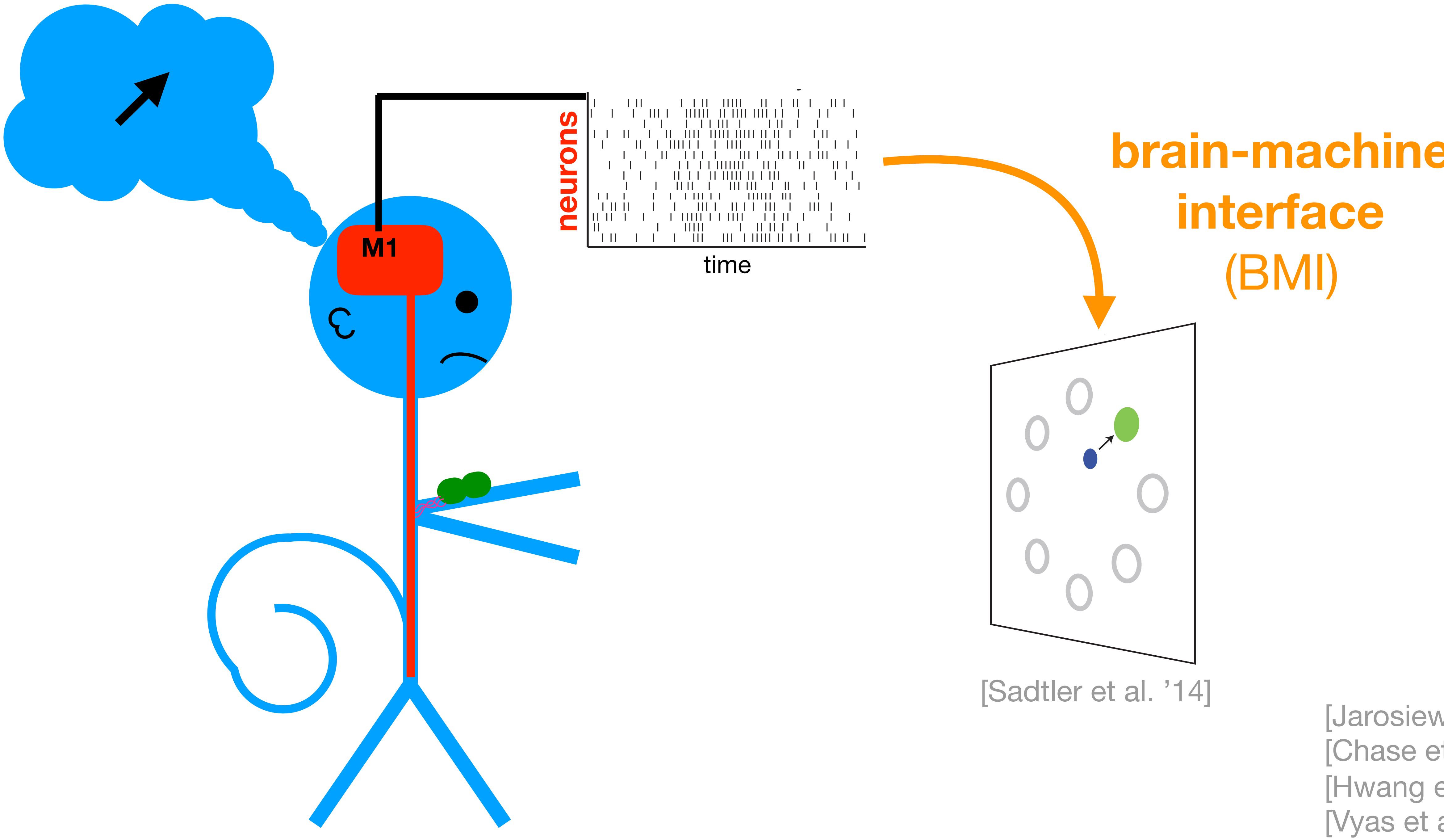

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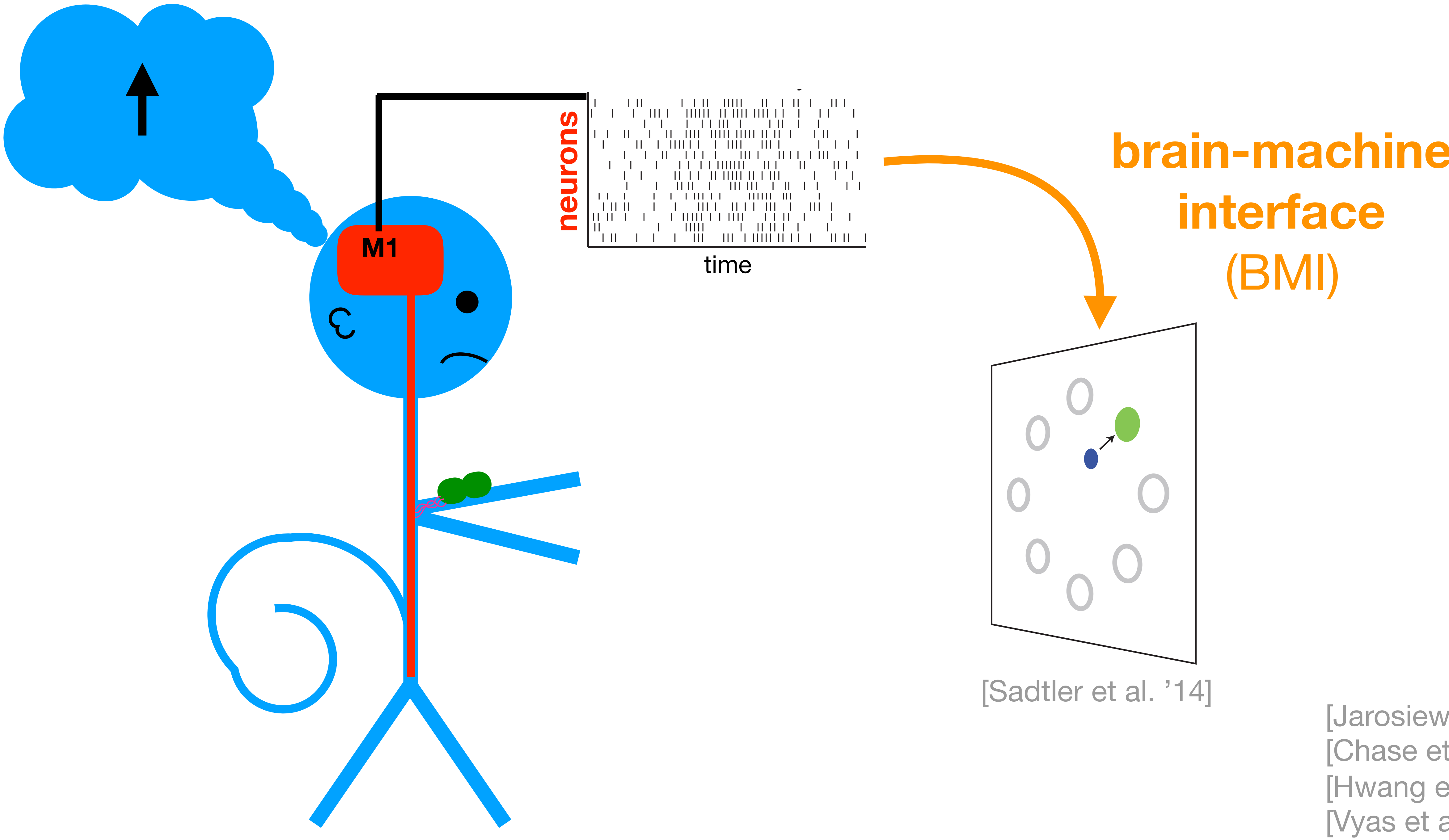

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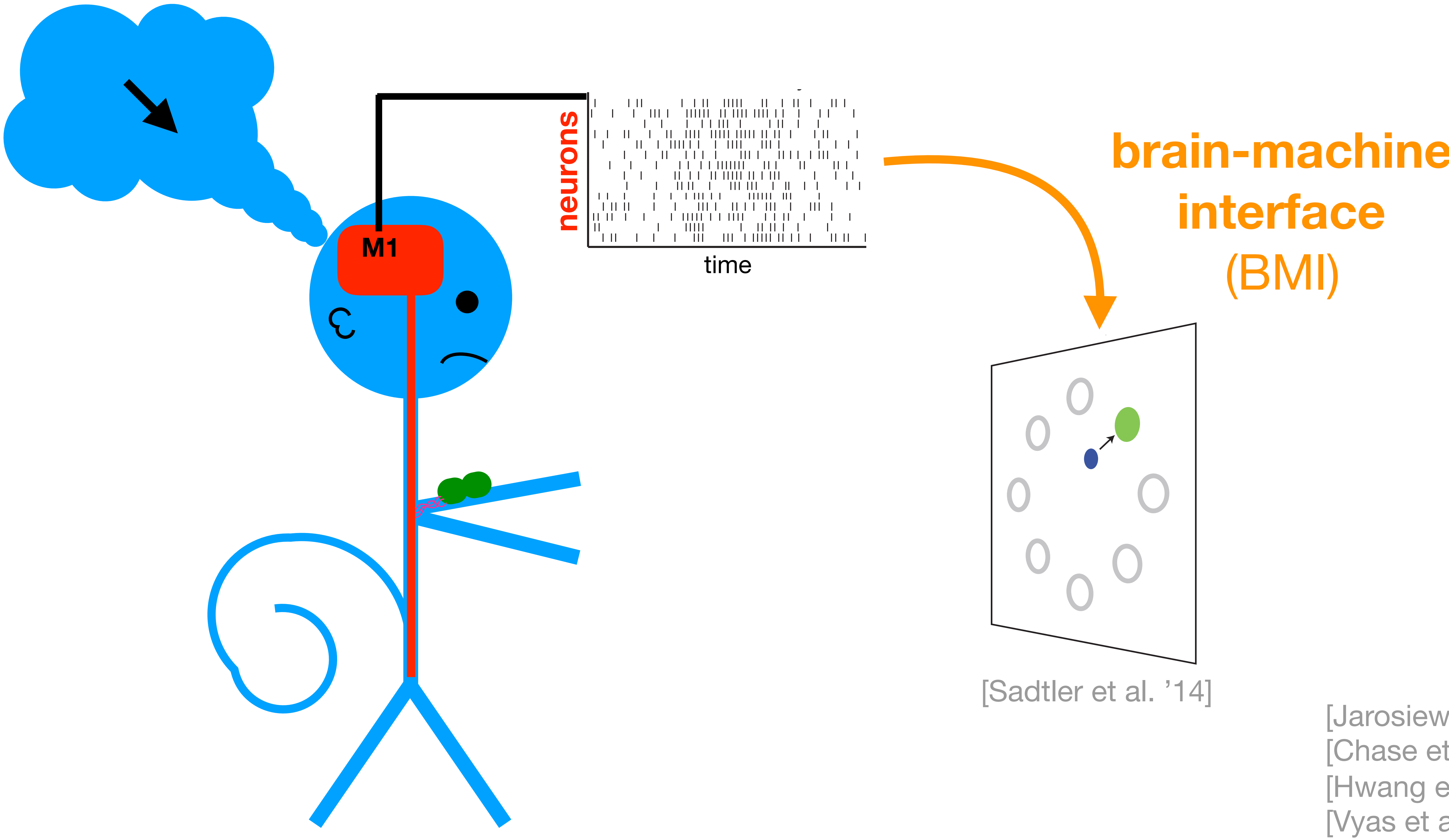

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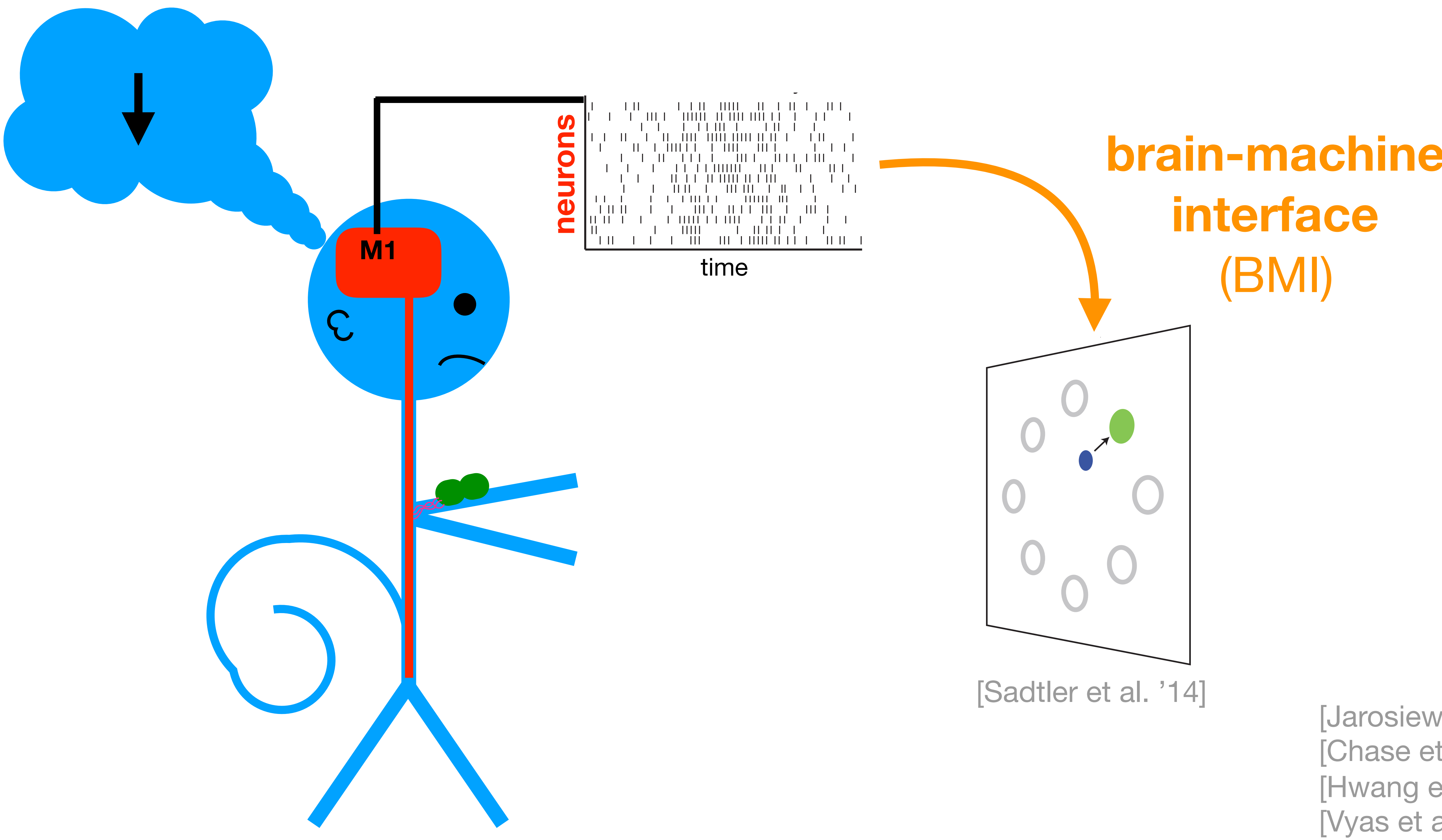

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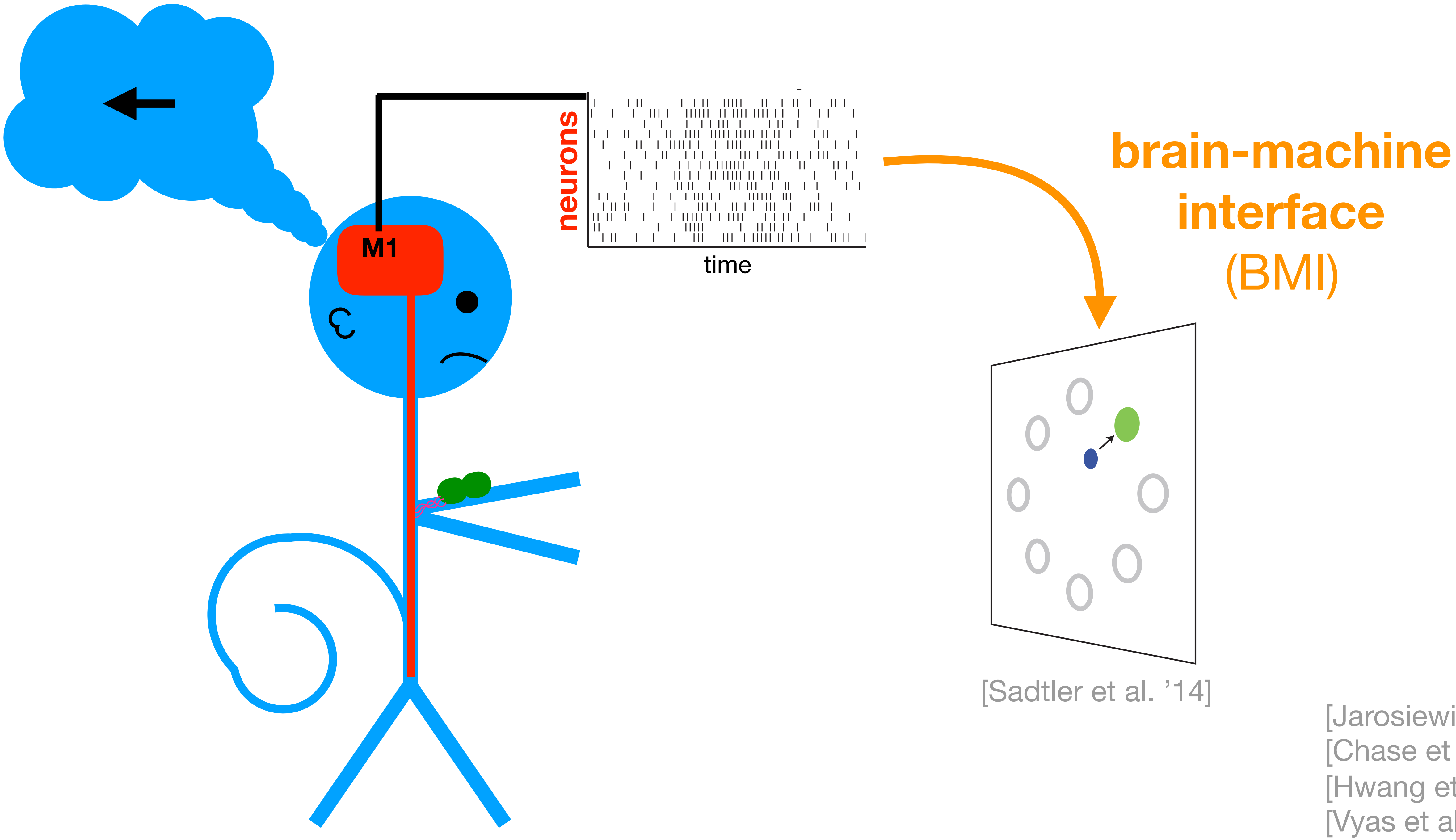

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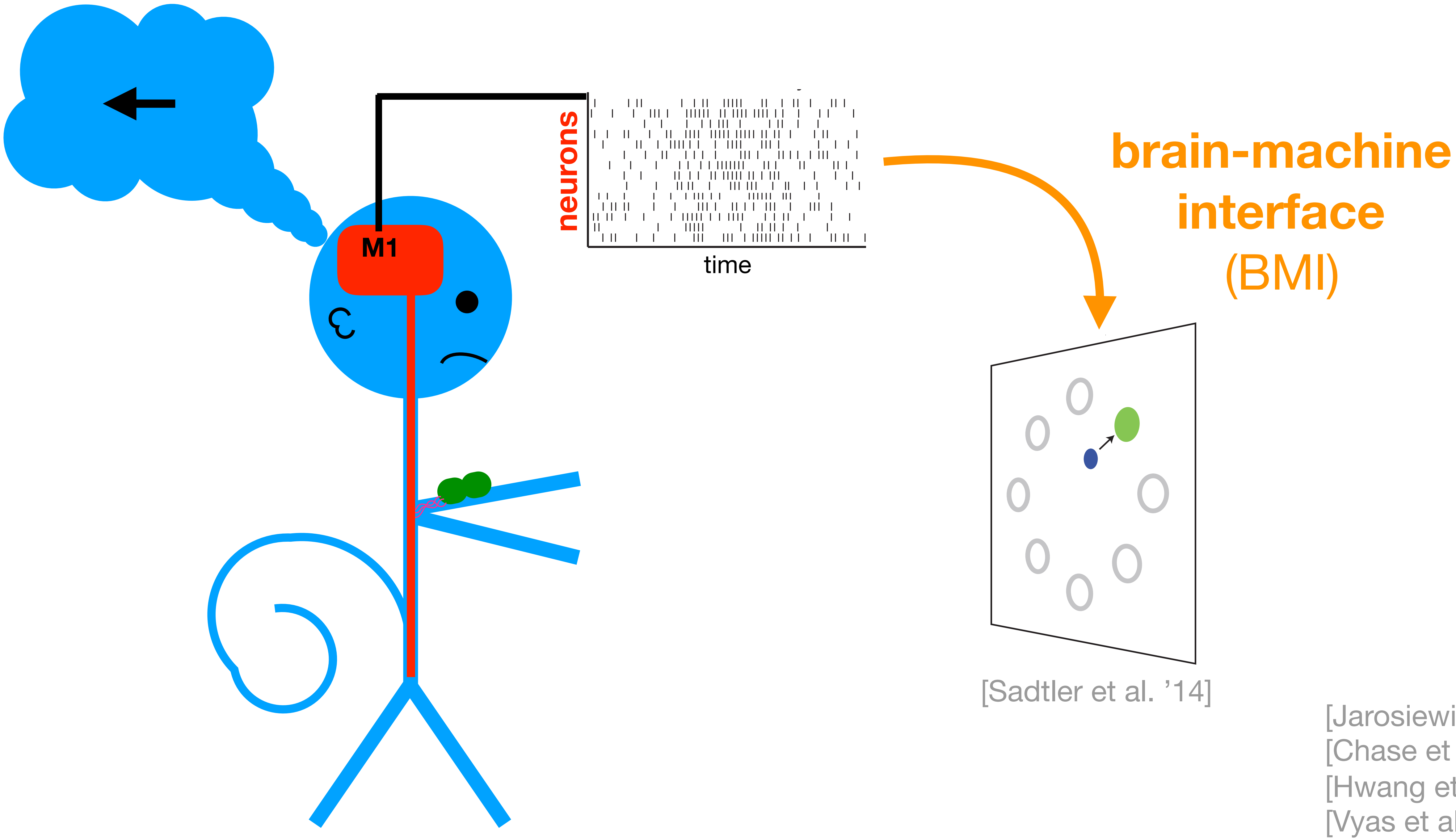

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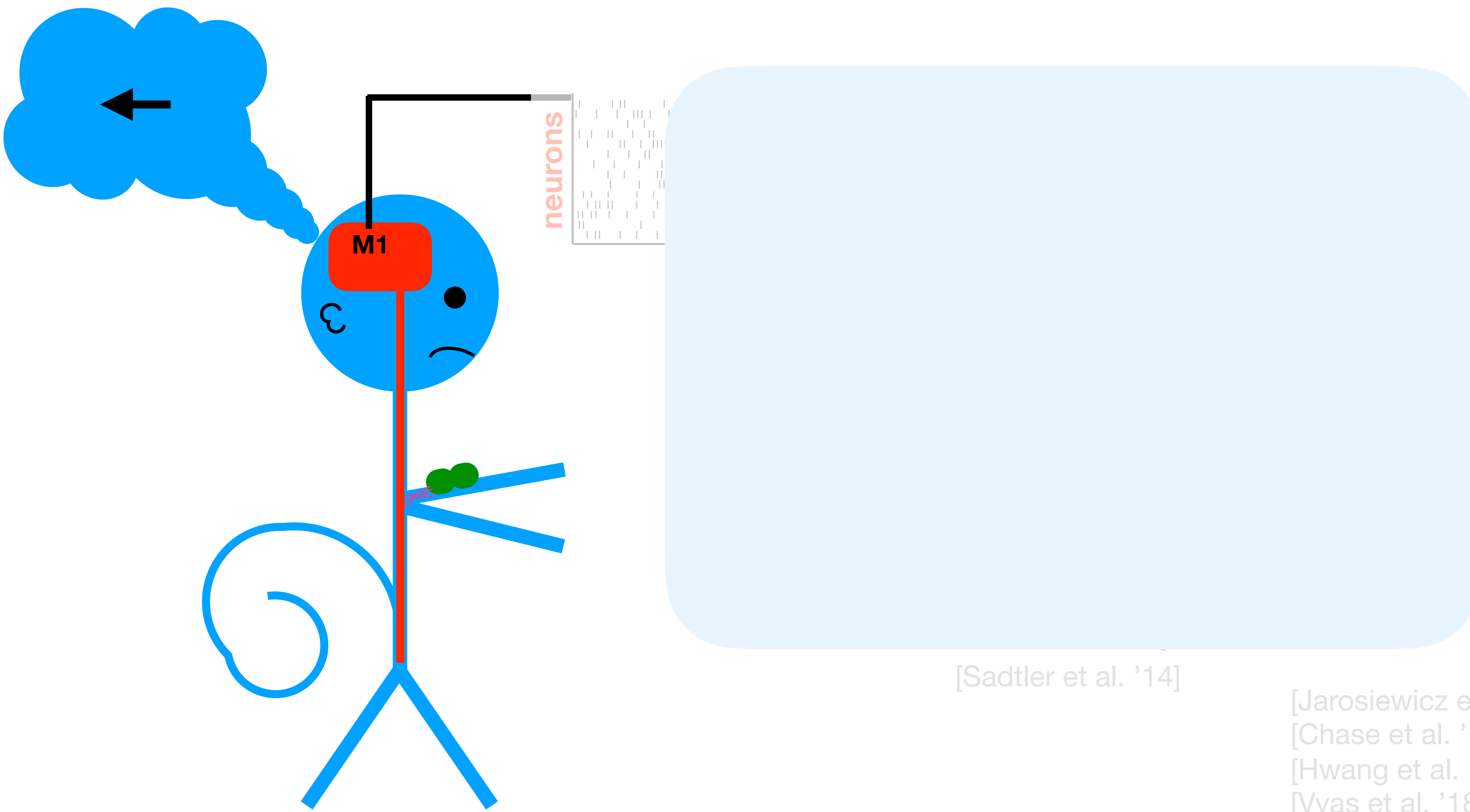

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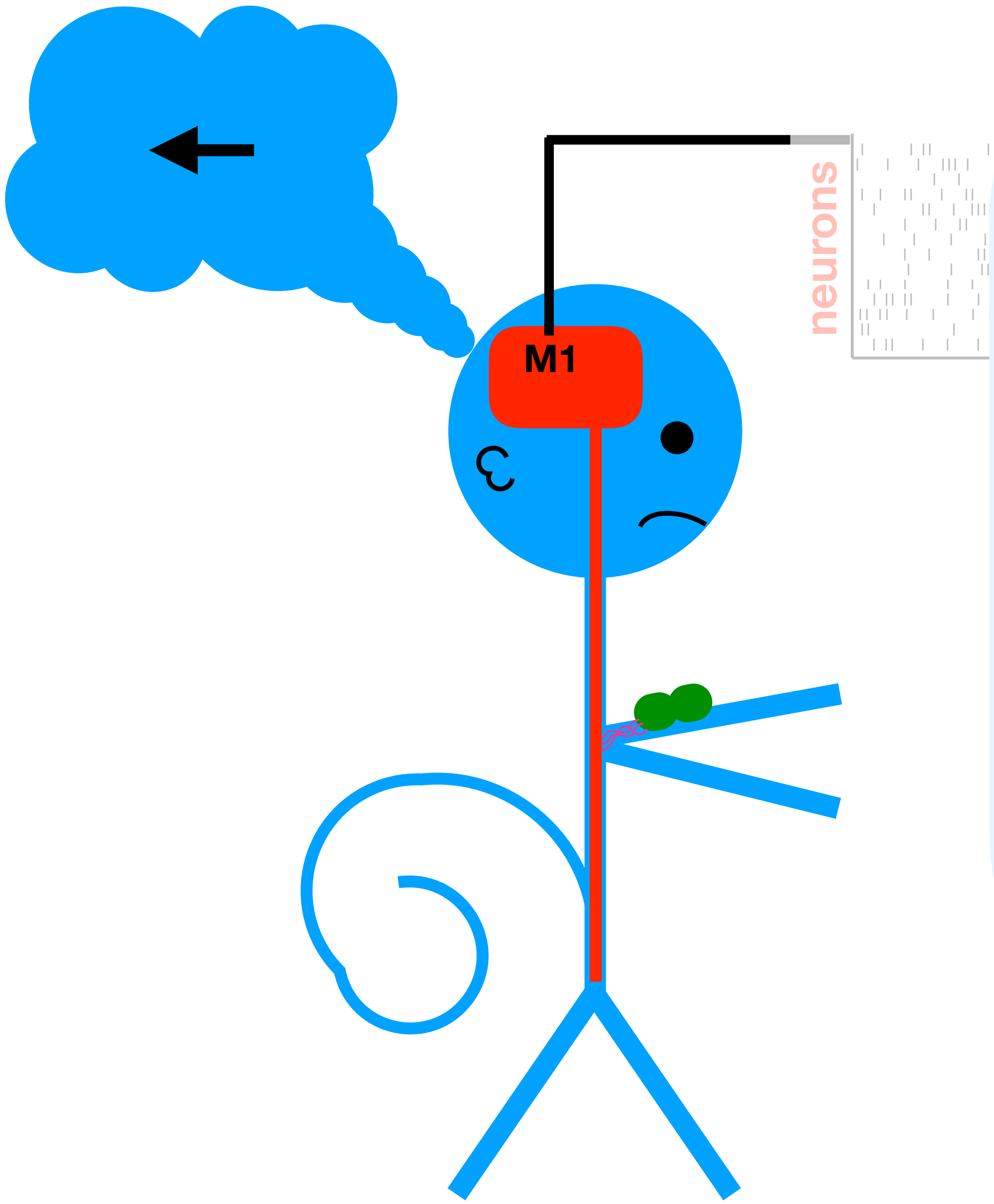

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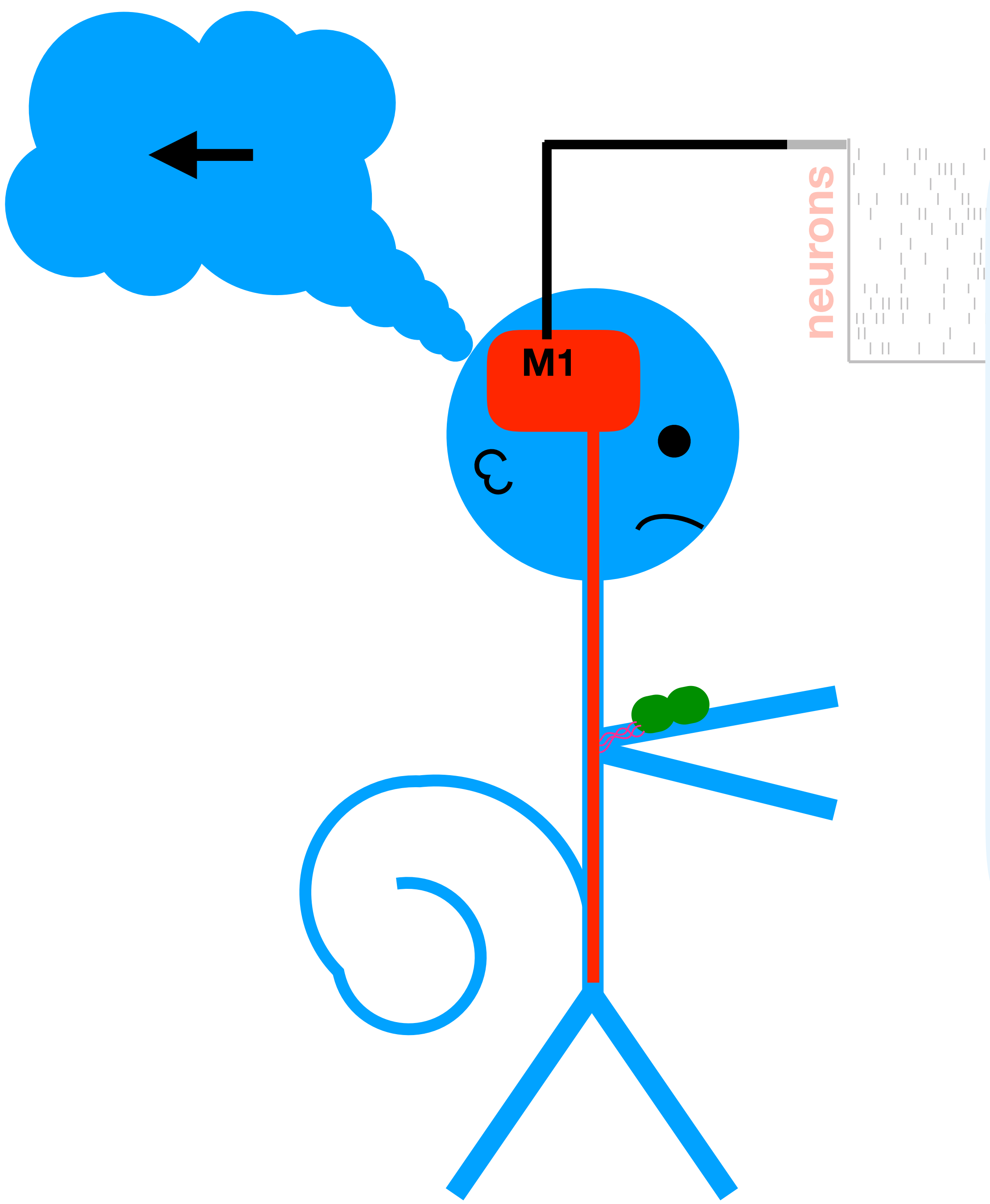

1) neural model

2) predictions for **population activity**

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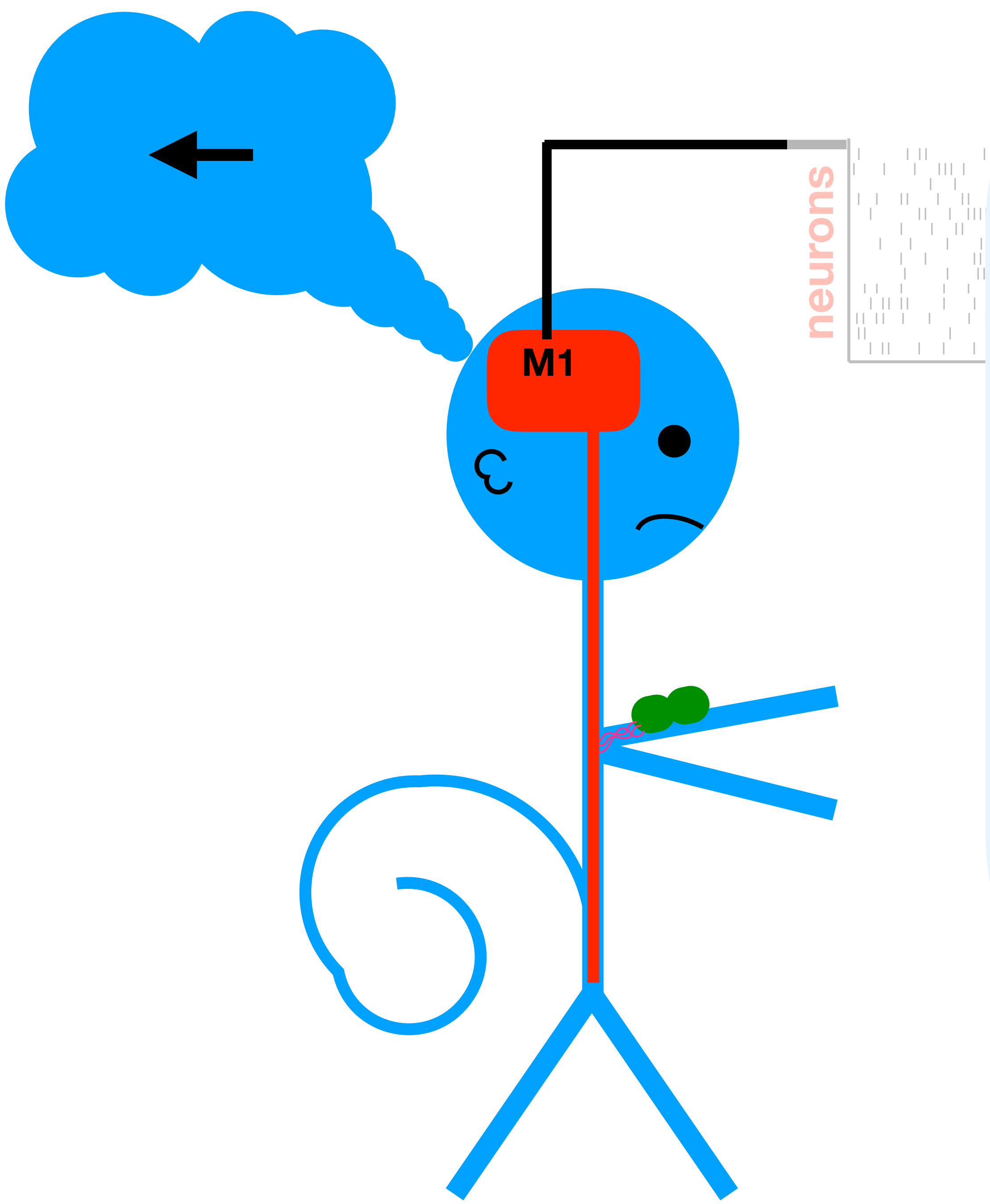

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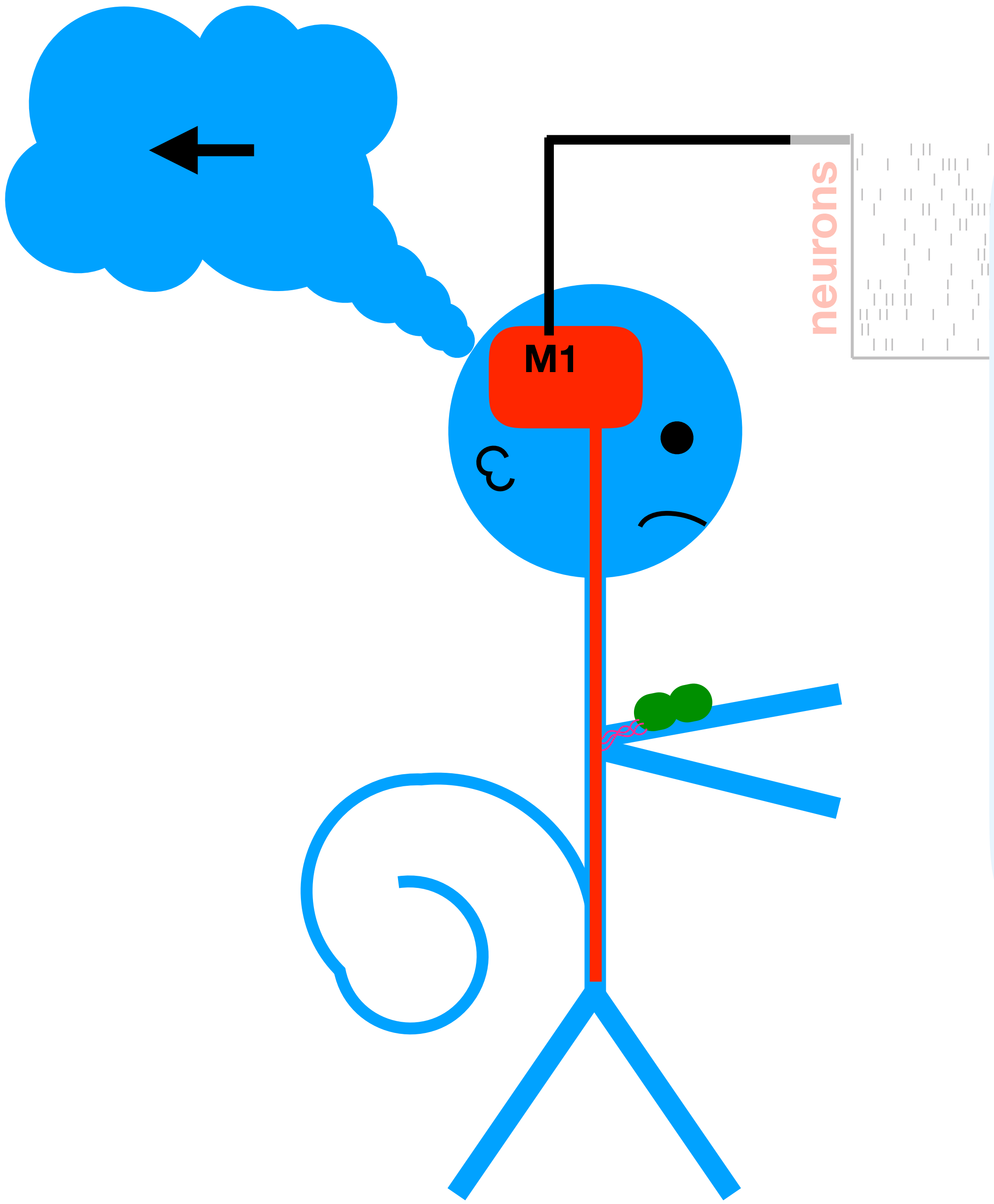

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test w/ data

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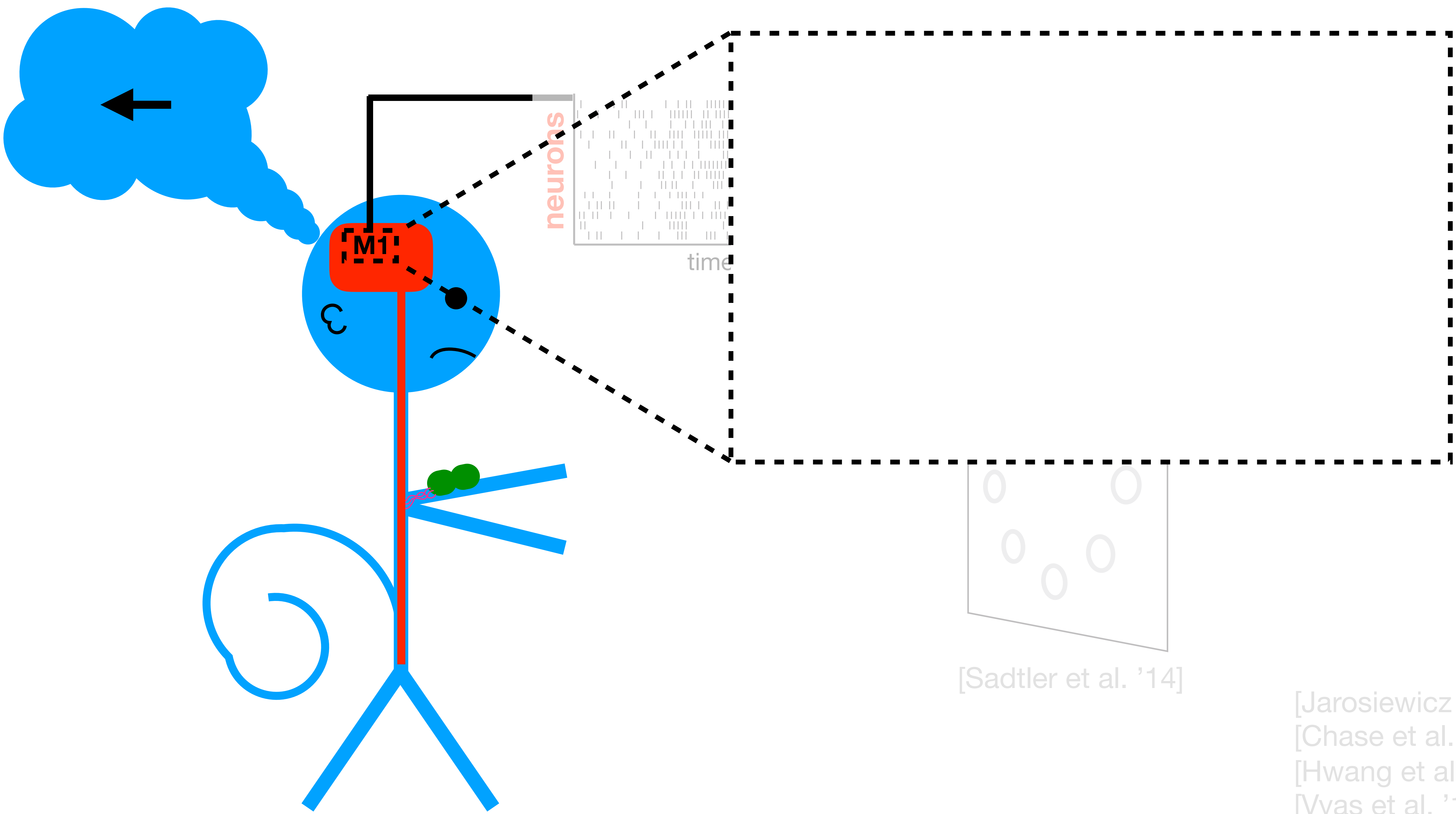

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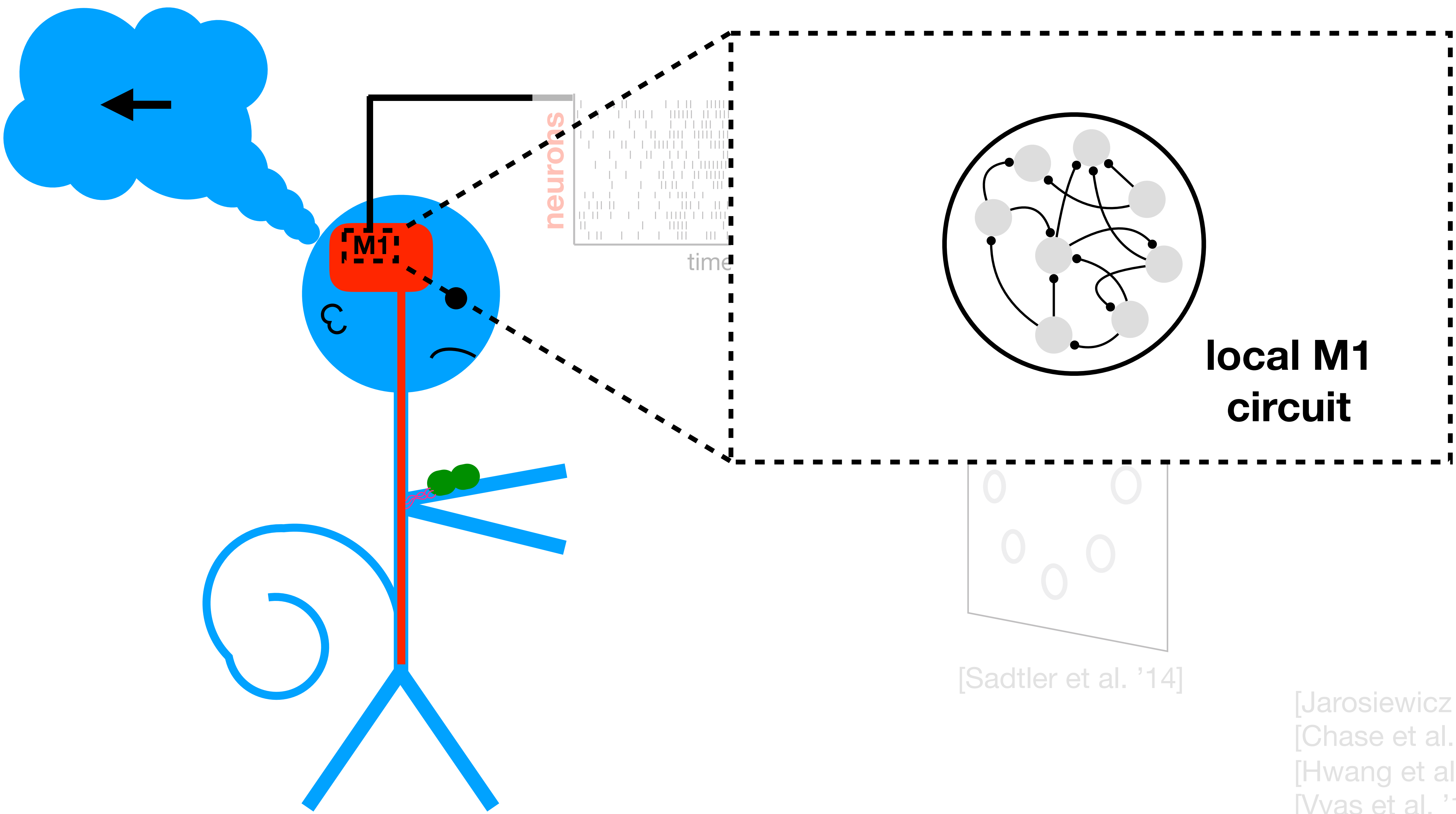

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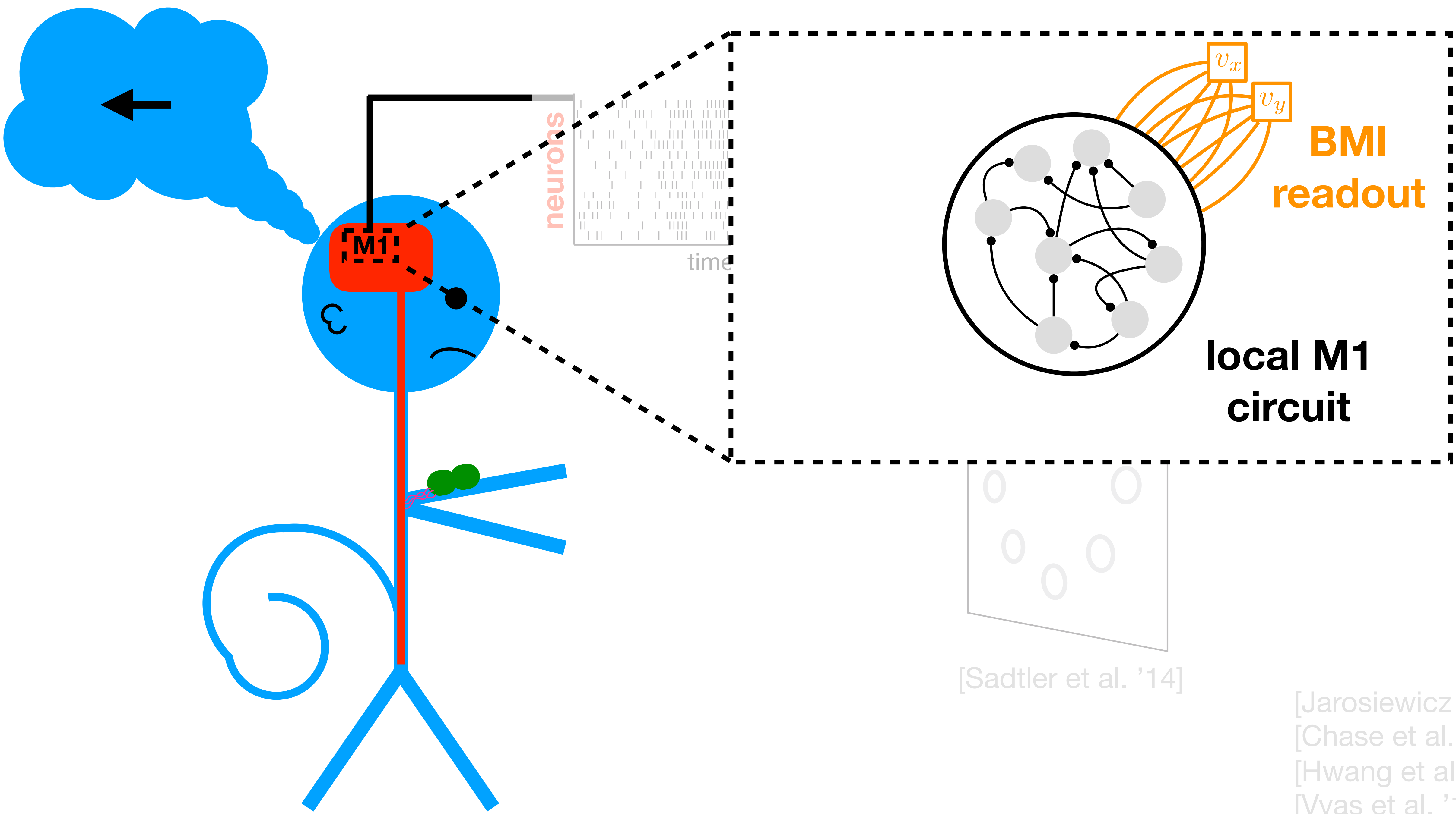

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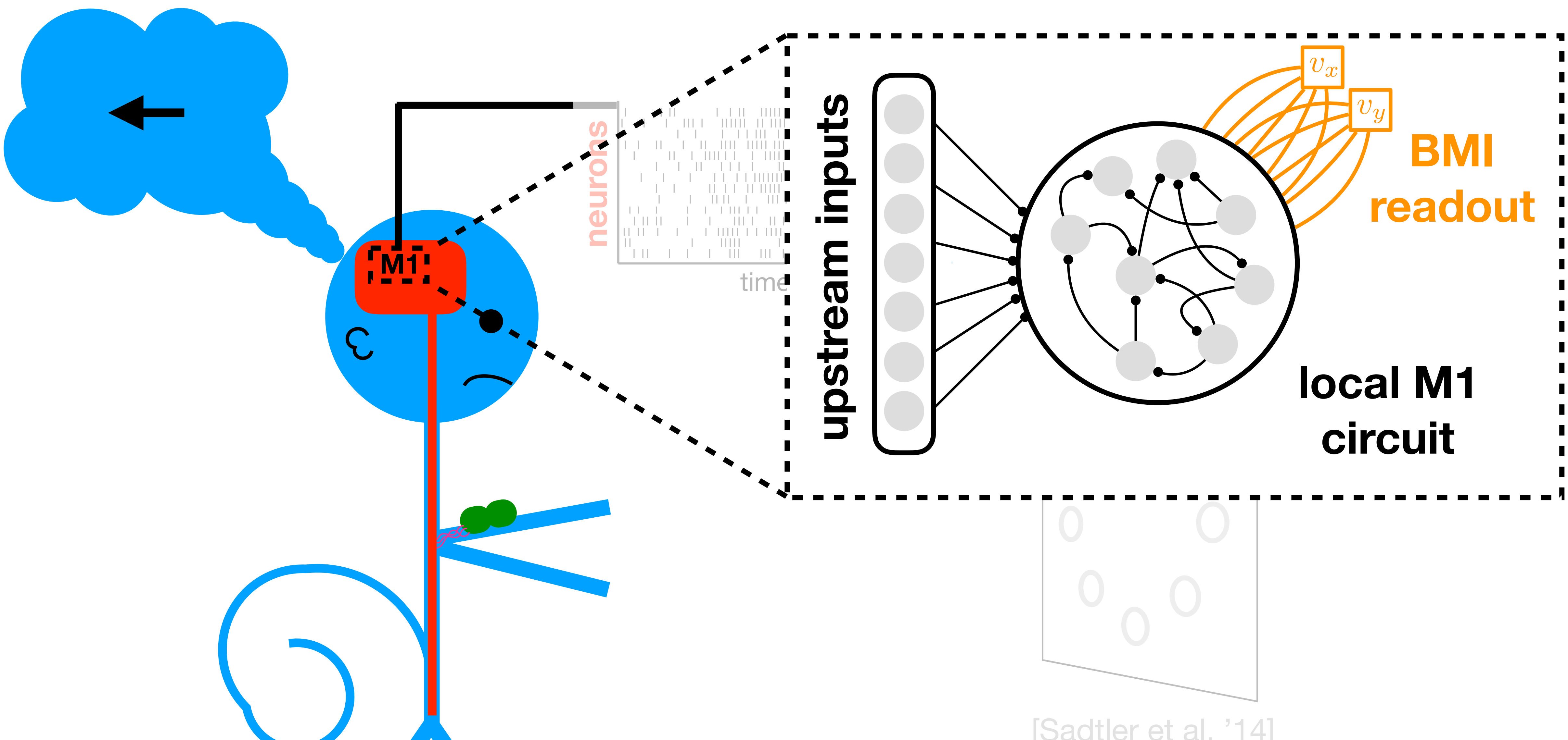

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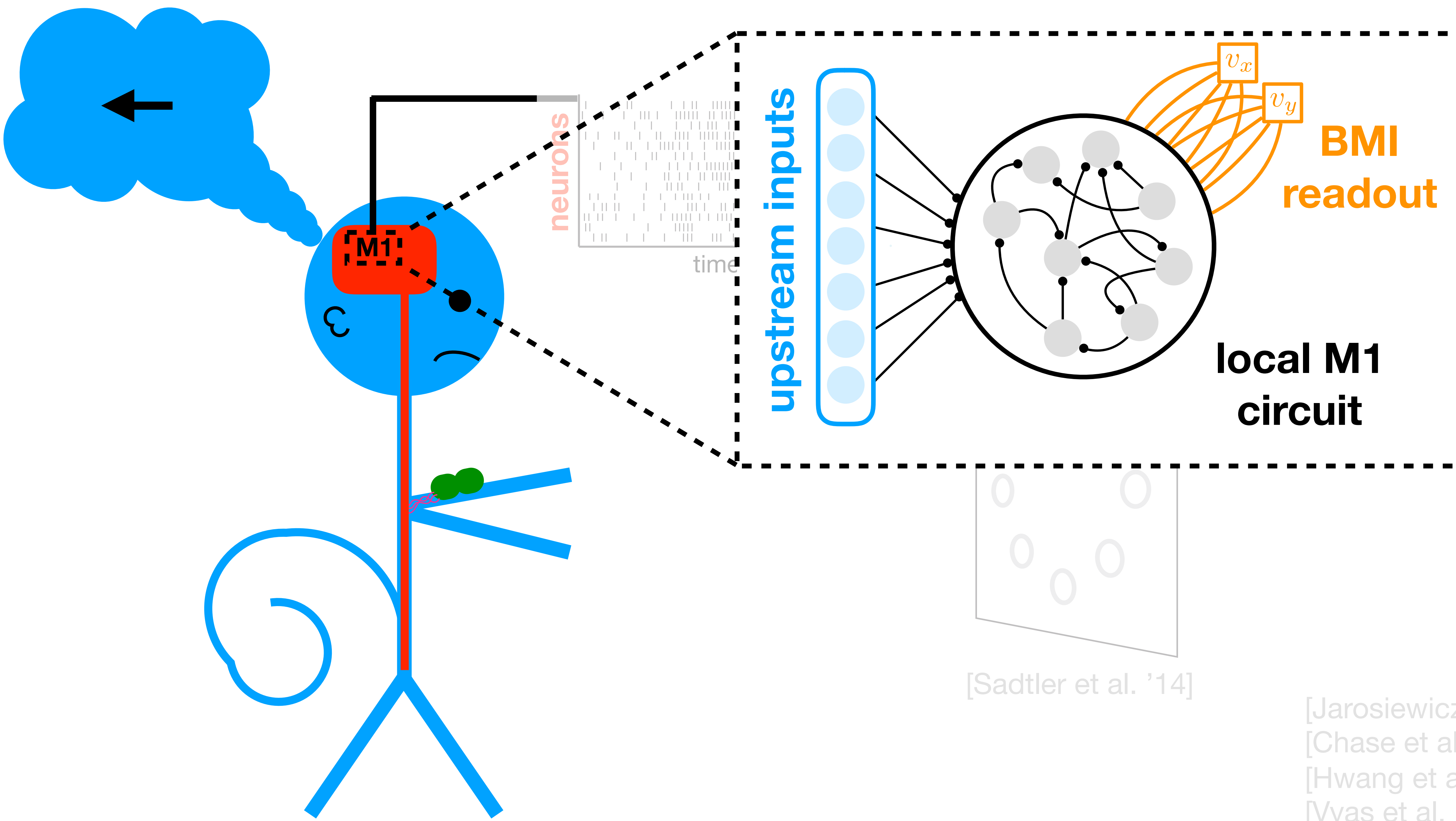

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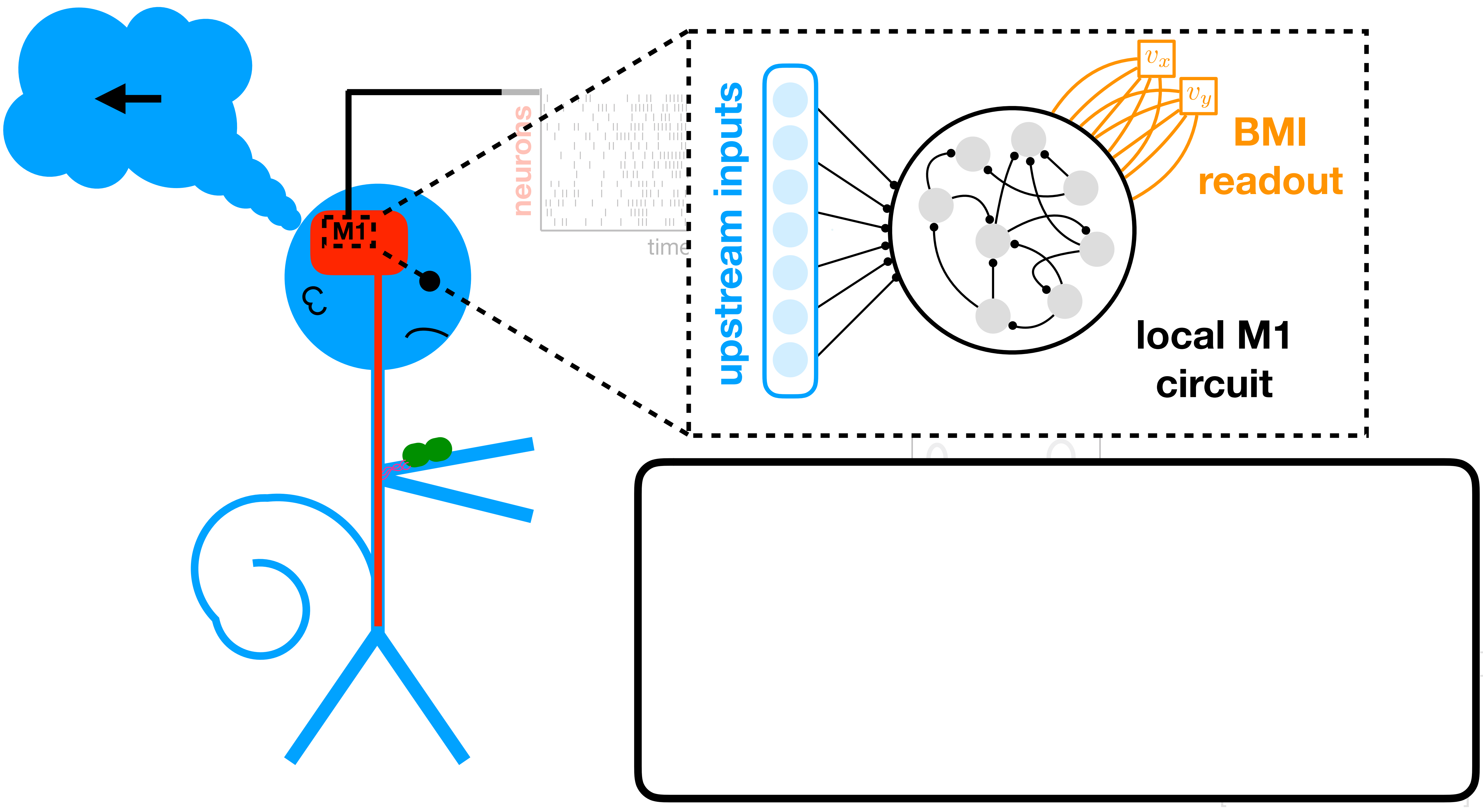

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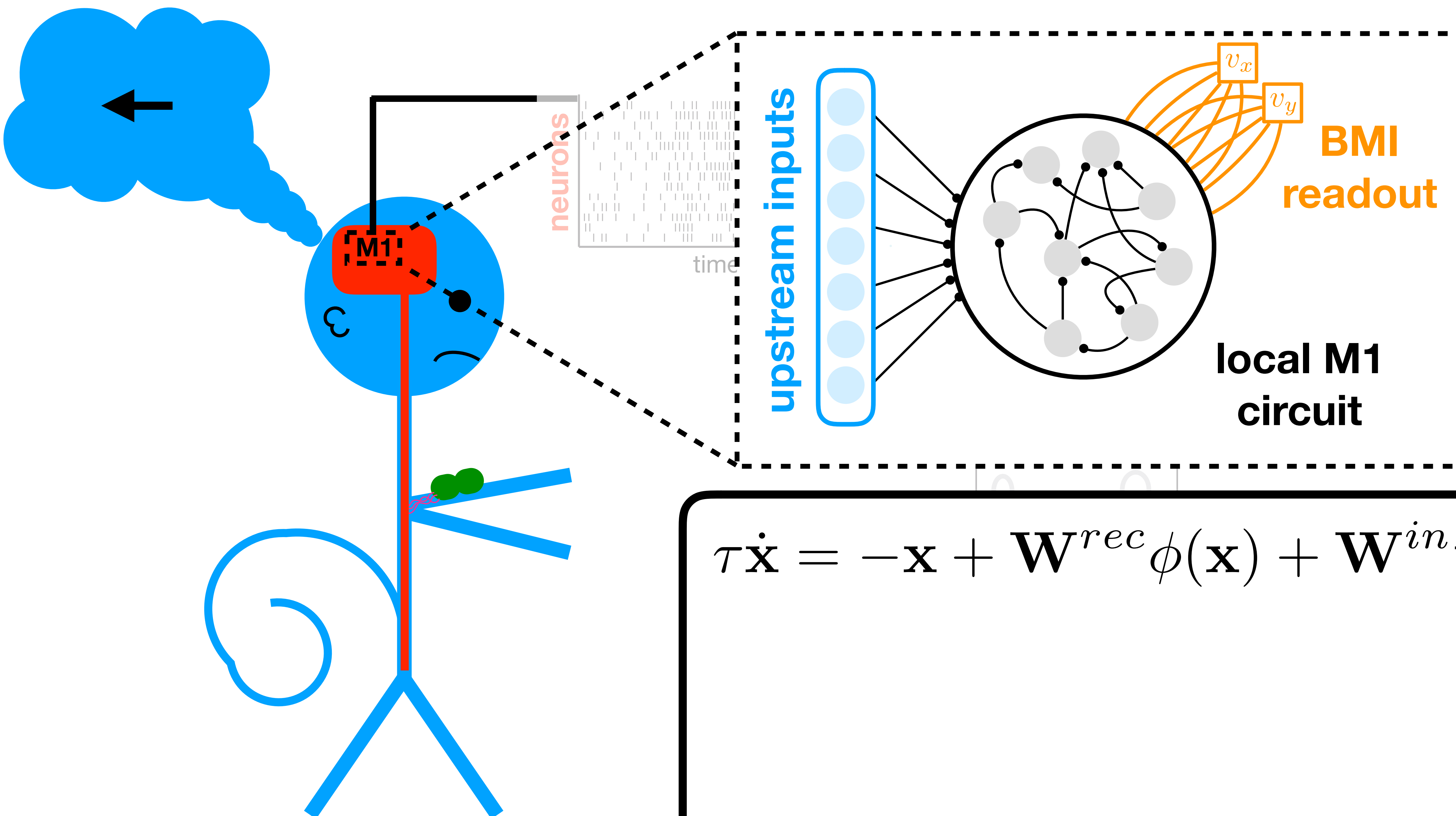

$$\tau \dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}^{rec}\phi(\mathbf{x}) + \mathbf{W}^{in}\mathbf{u}$$

BMI learning

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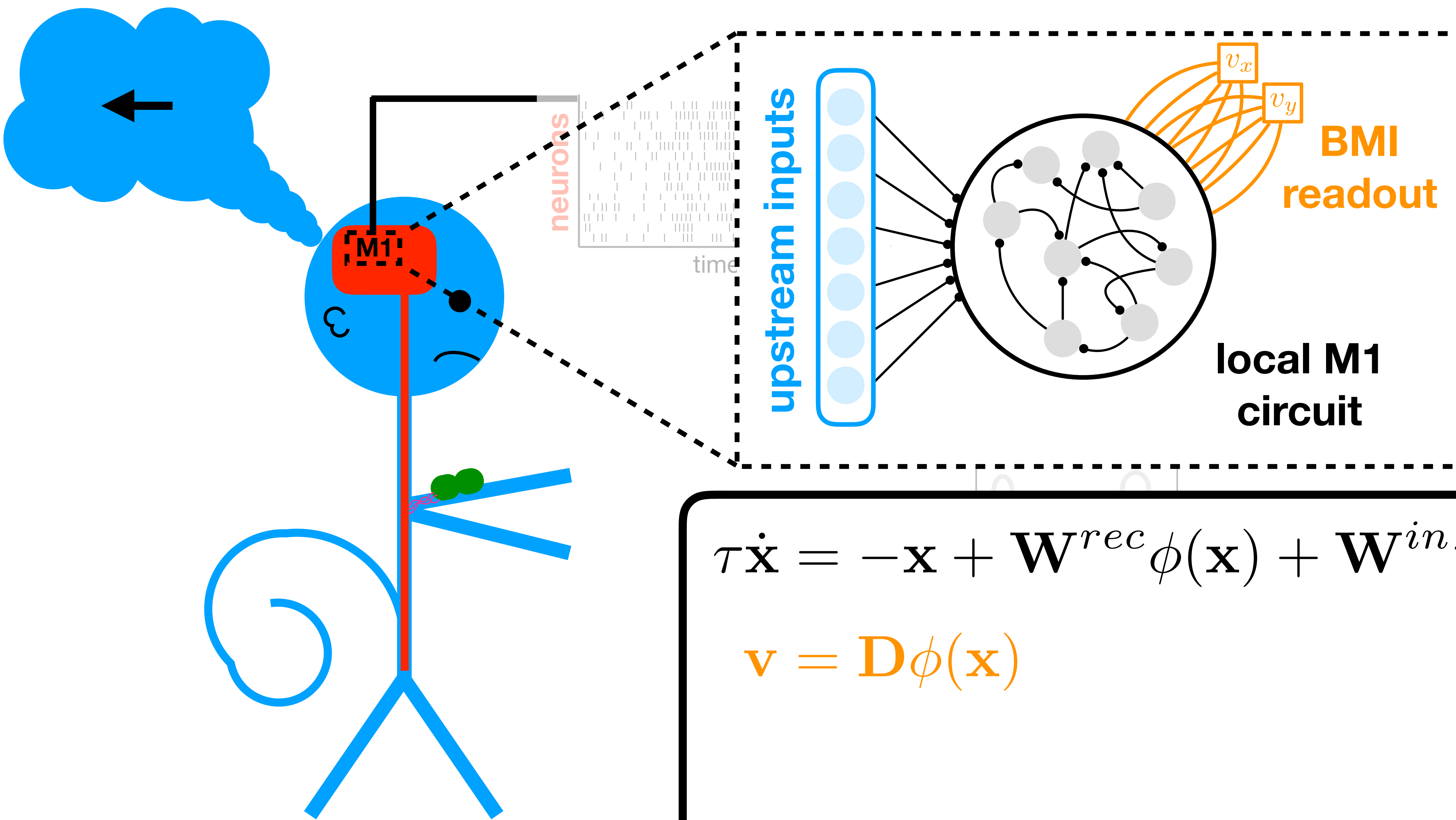

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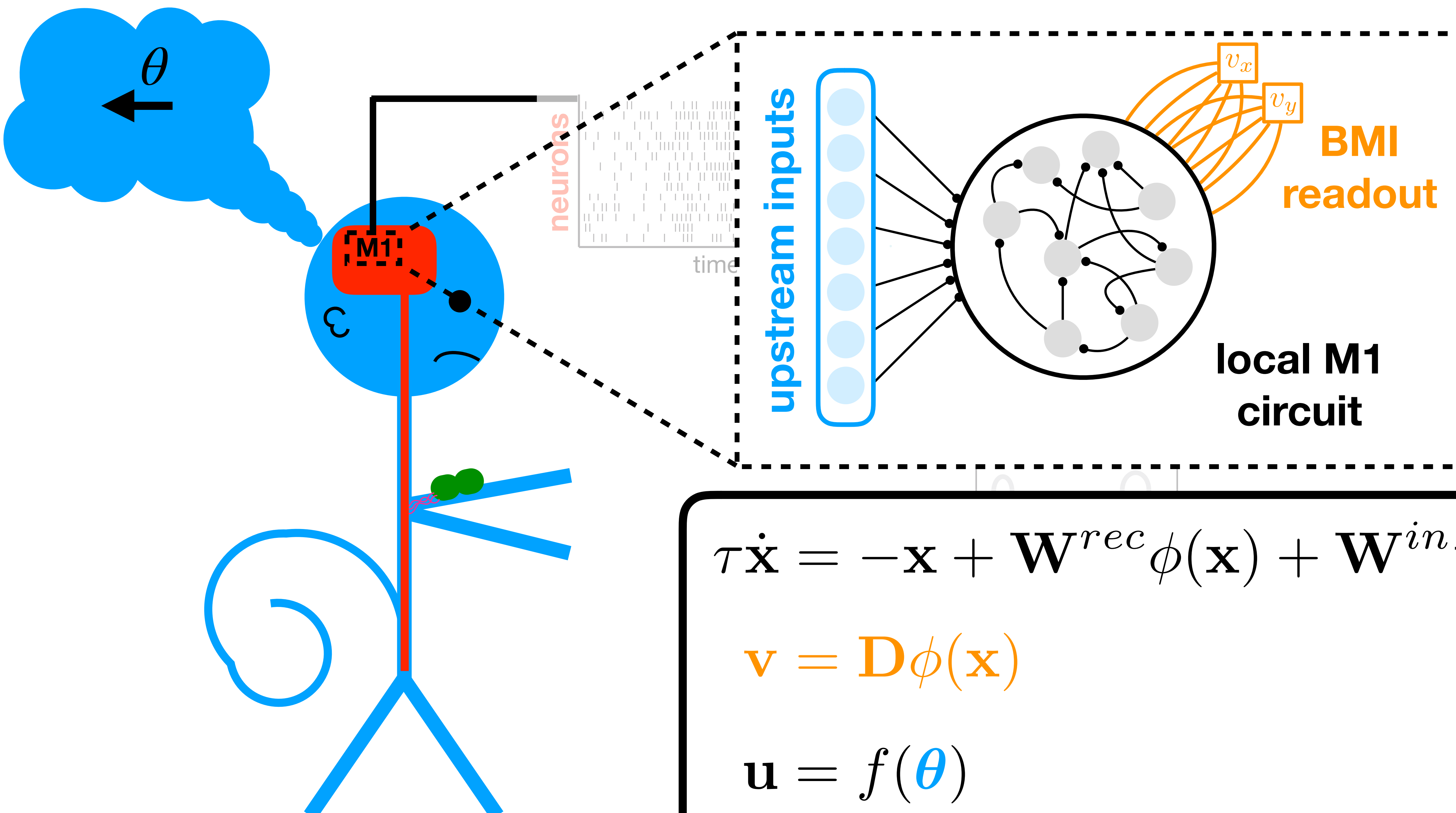

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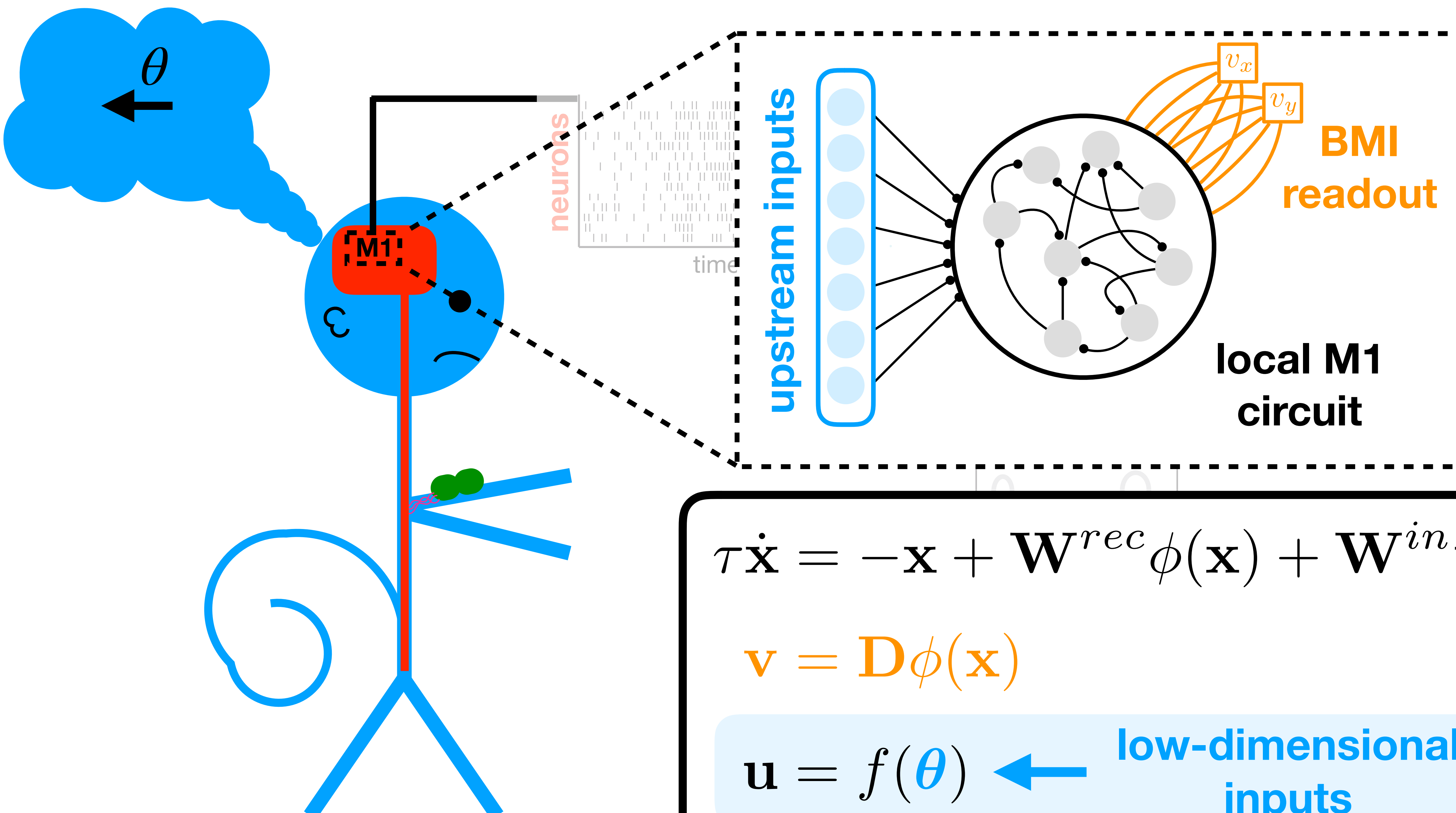

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← low-dimensional inputs

“re-aiming”

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Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_{\theta} = \mathbf{M}\boldsymbol{\theta}$

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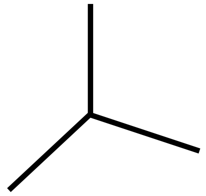

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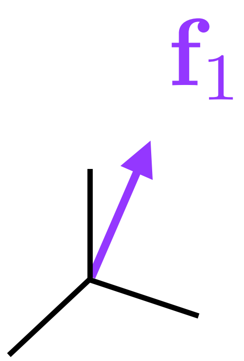

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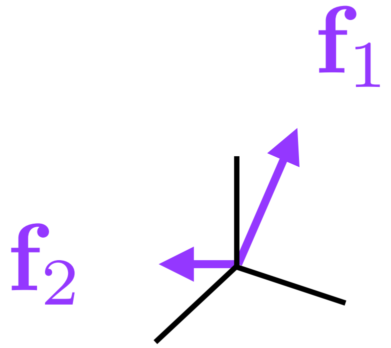

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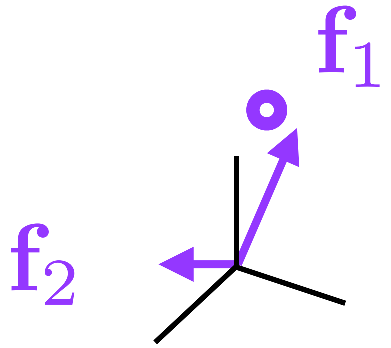
[θ *constant*] $\Rightarrow \mathbf{x}_\theta(t) = \mathbf{f}_1\theta_1 + \mathbf{f}_2\theta_2$



“re-aiming”

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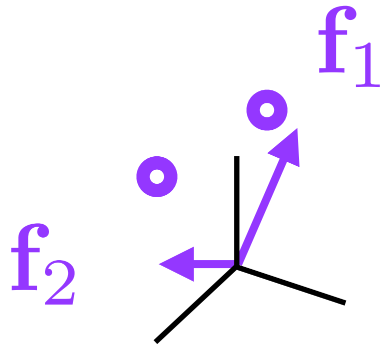

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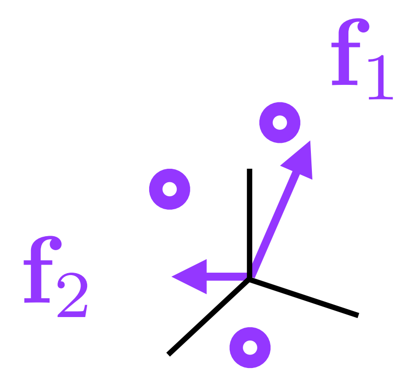

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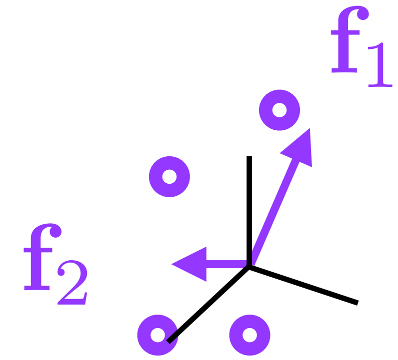

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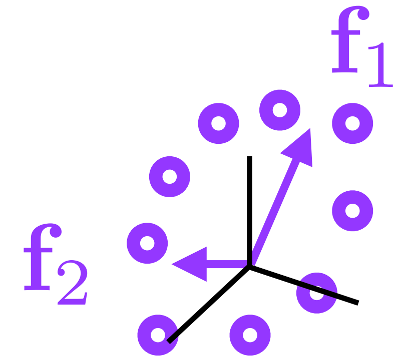

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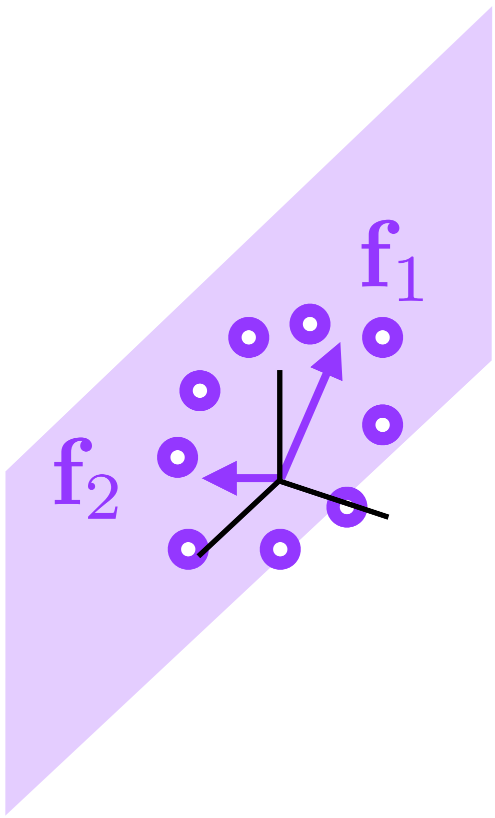

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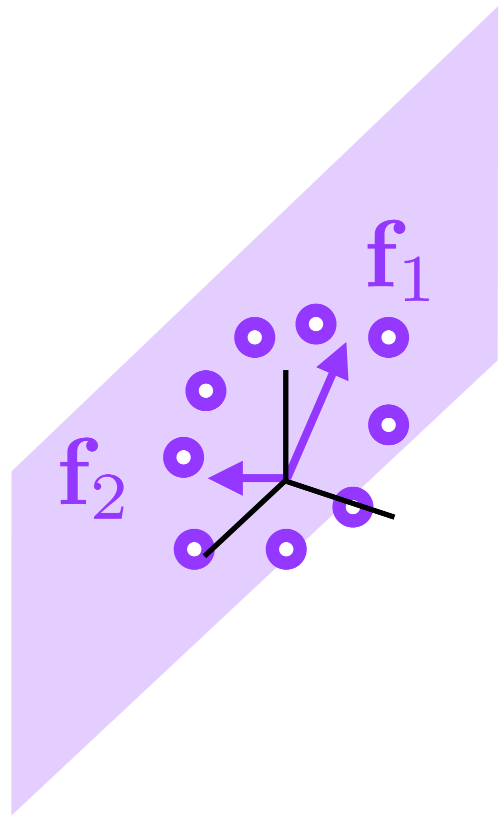

(1) low-dimensional activity } **“re-aiming”**

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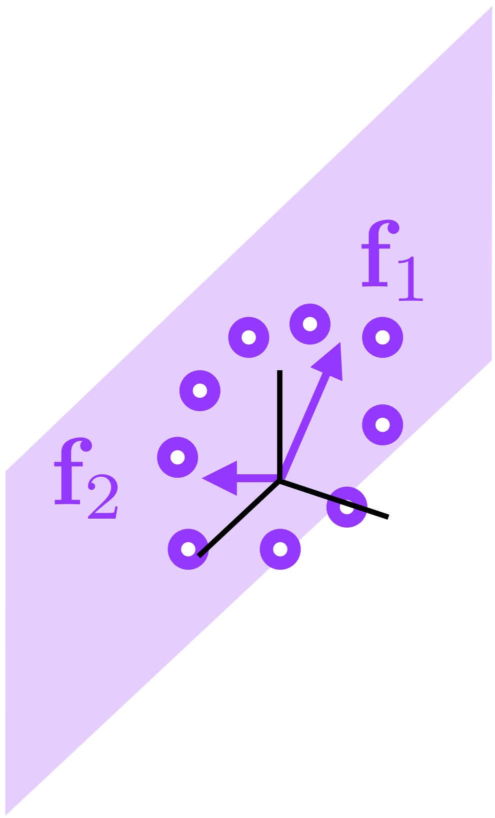

[Sadtler et al. ’14]
[Gao & Ganguli ’17]
[Golub et al. ’18]
[Hennig et al. ’18]

(1) low-dimensional activity

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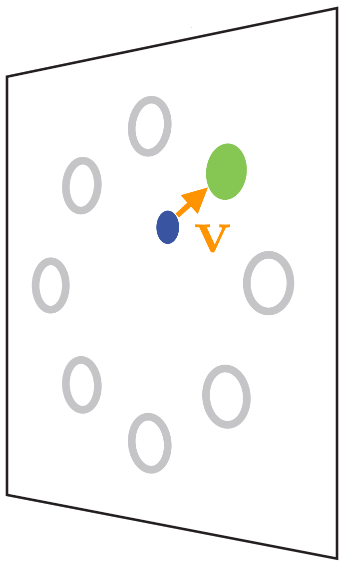

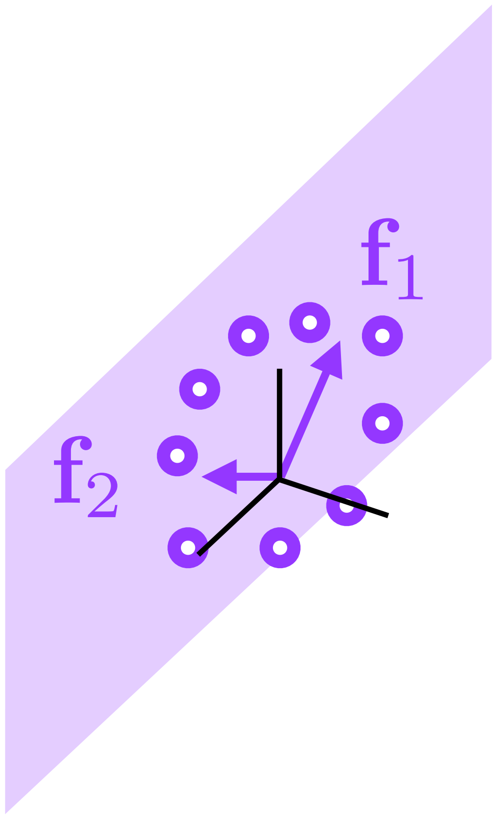

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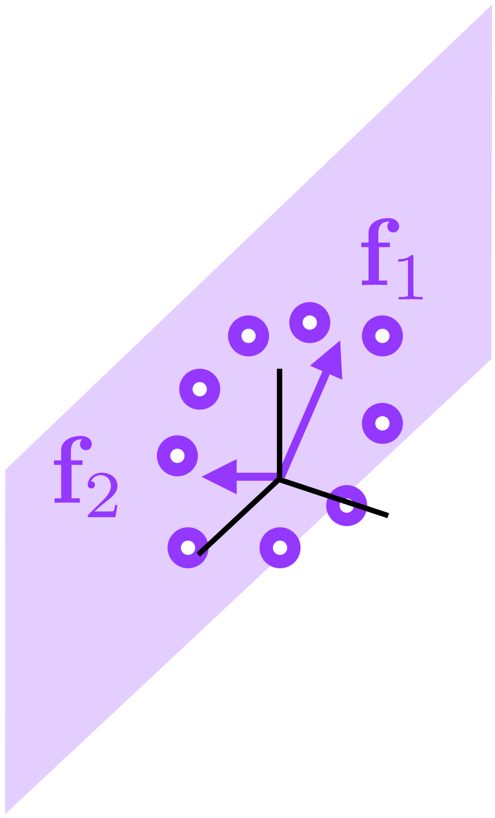
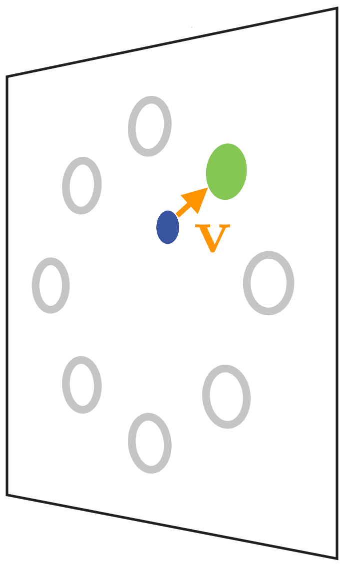


$$\mathbf{v}(t) = \mathbf{D}\mathbf{x}_\theta(t)$$



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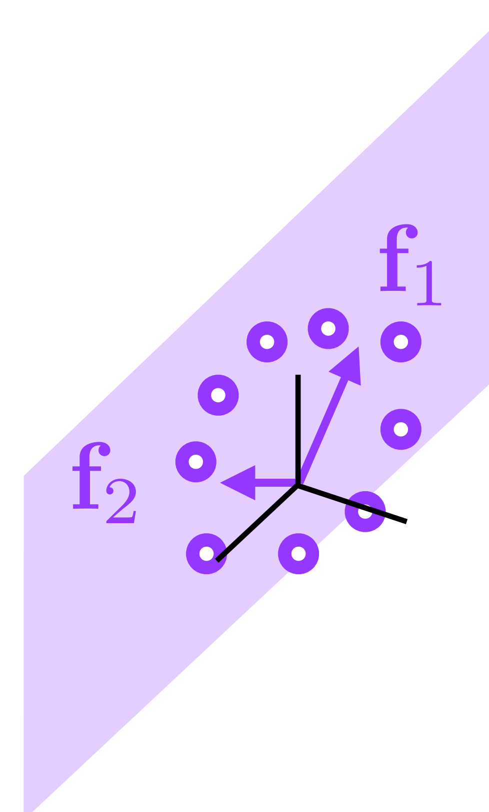

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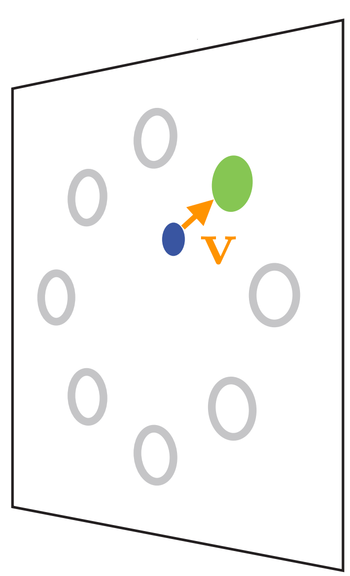

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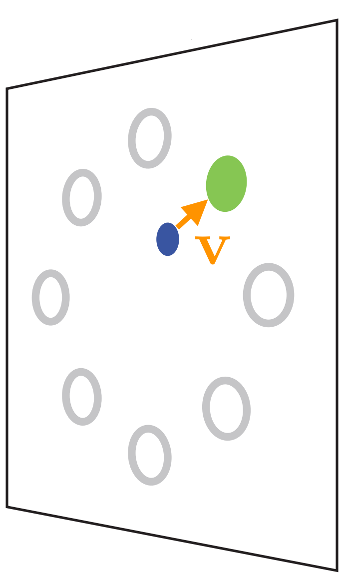

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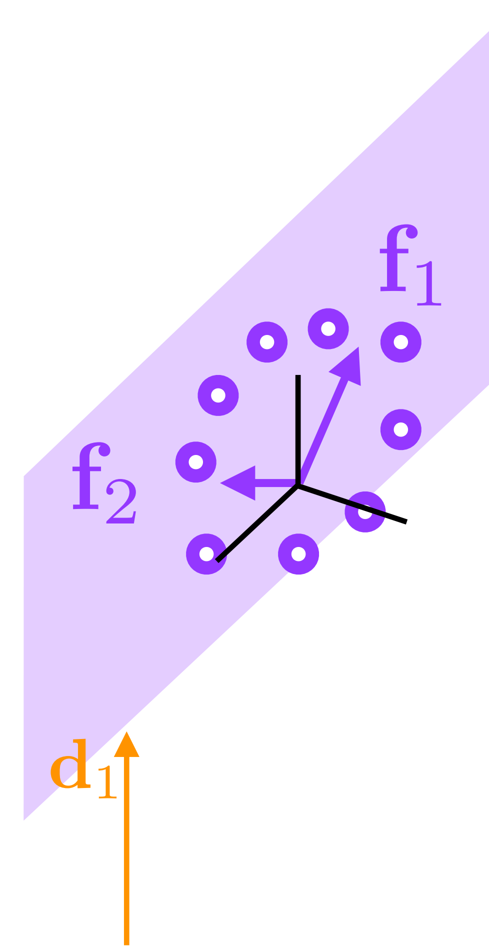

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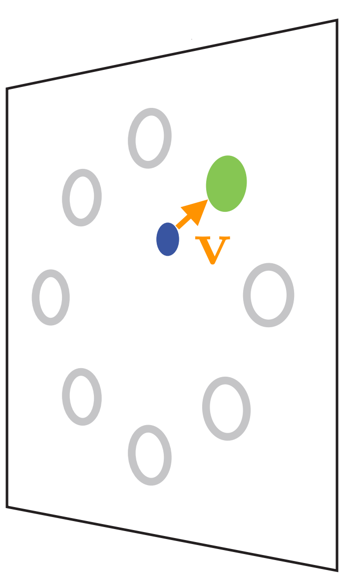

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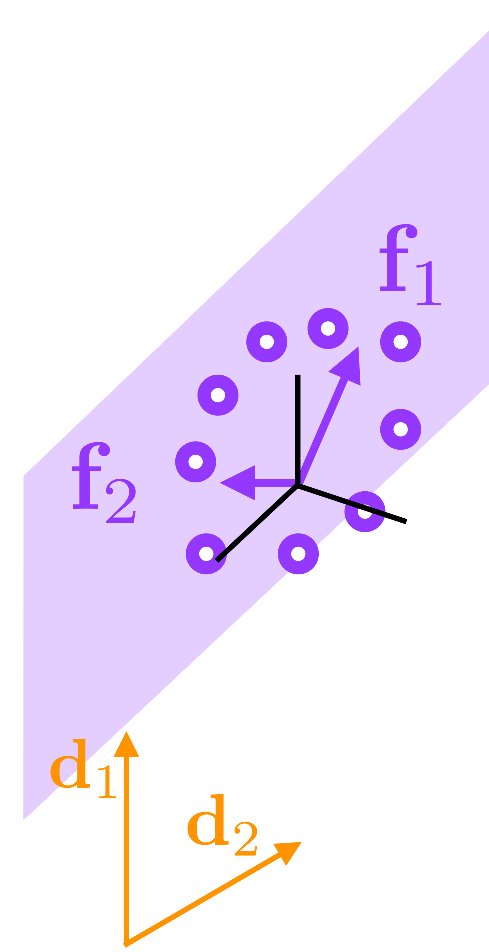

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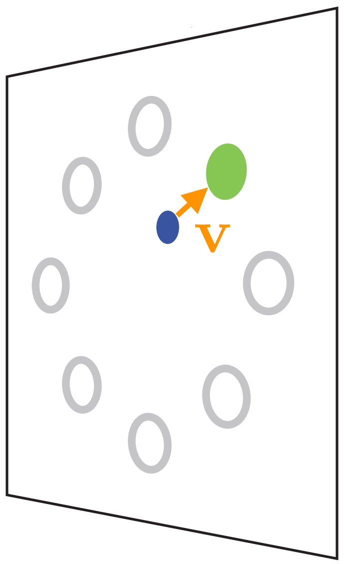

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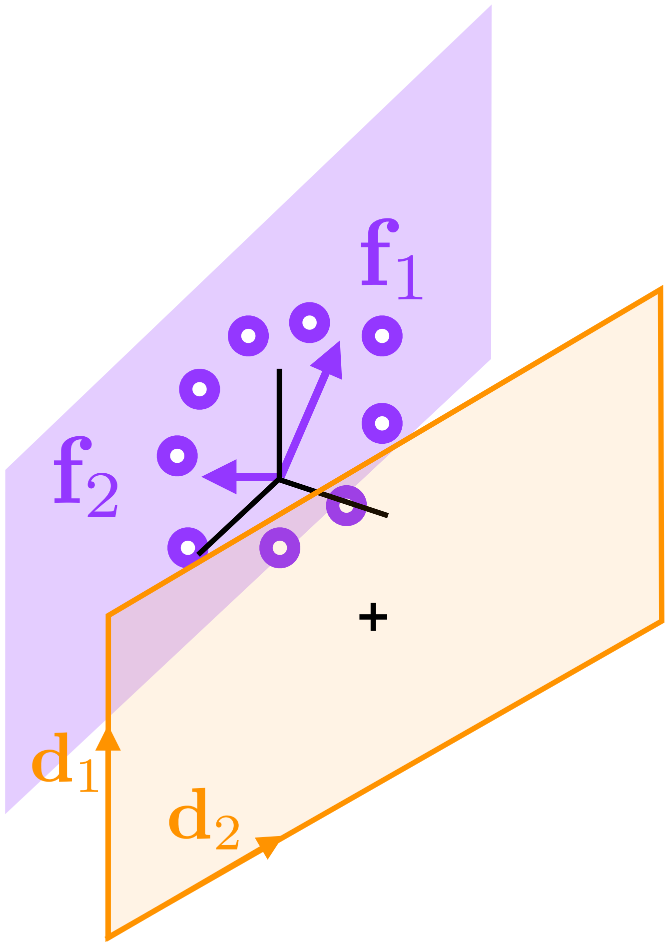

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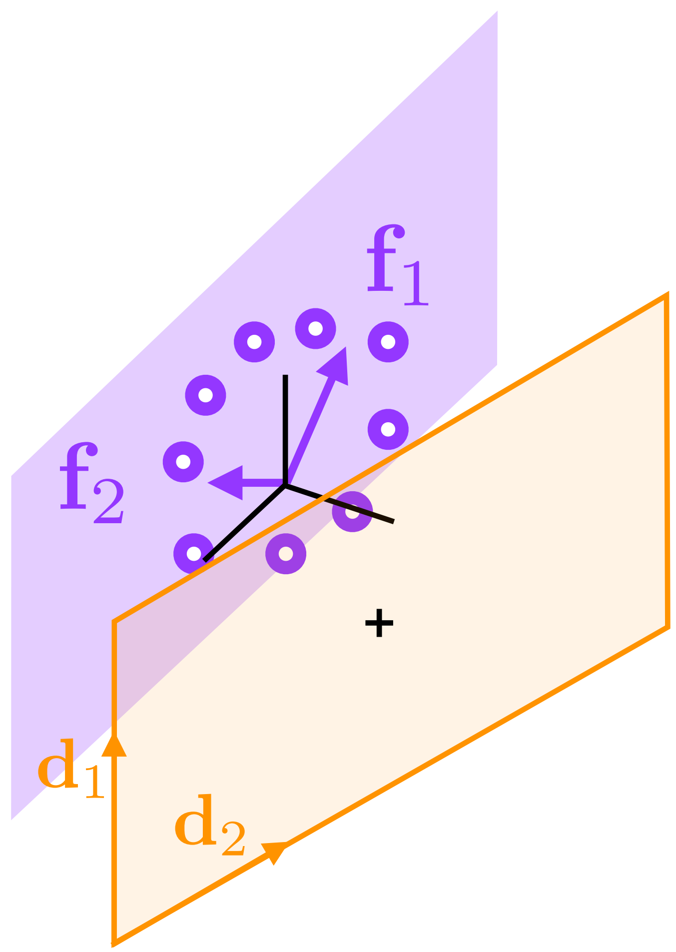

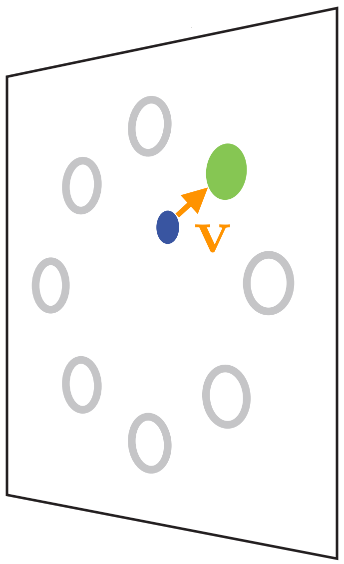

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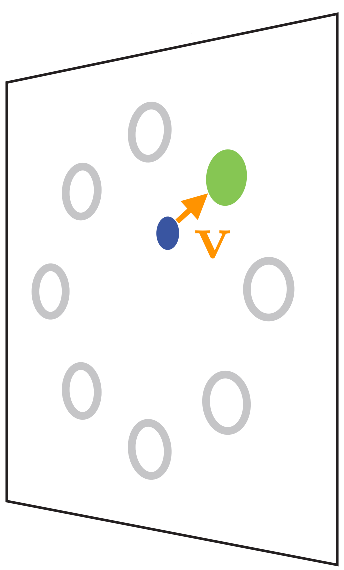

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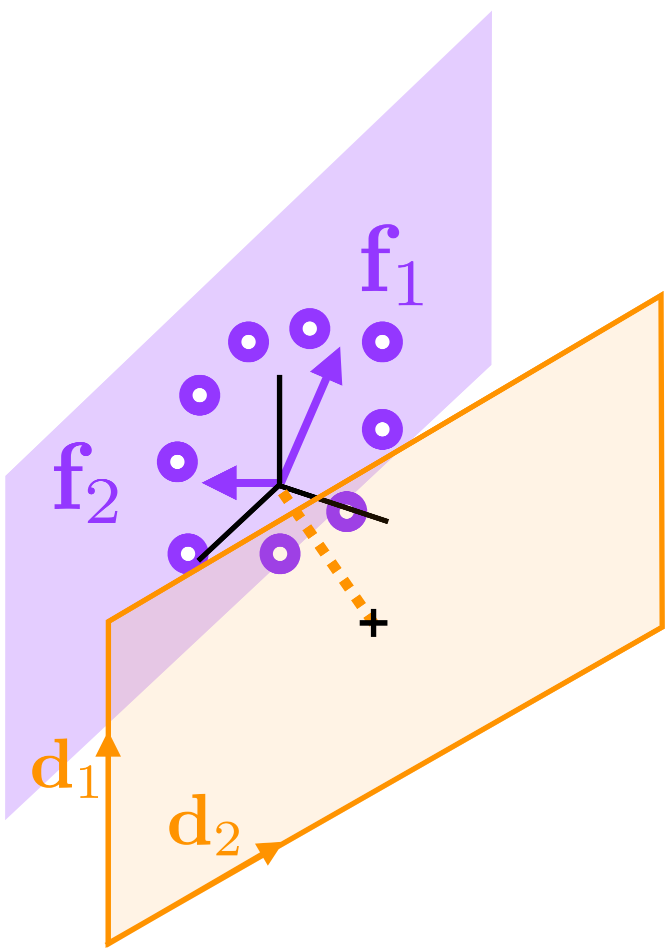

(1) low-dimensional activity

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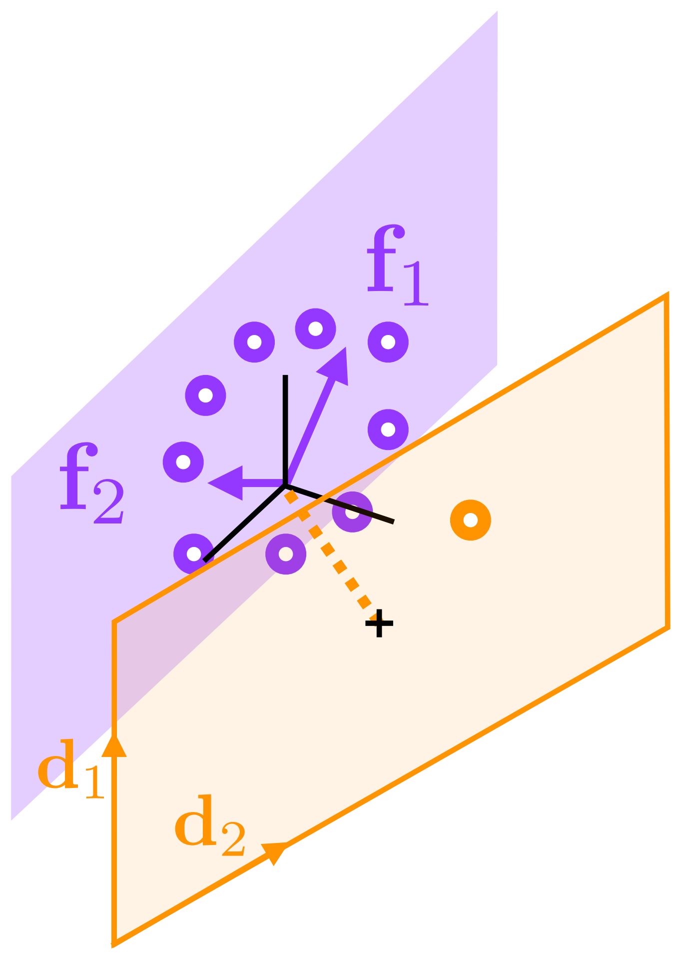

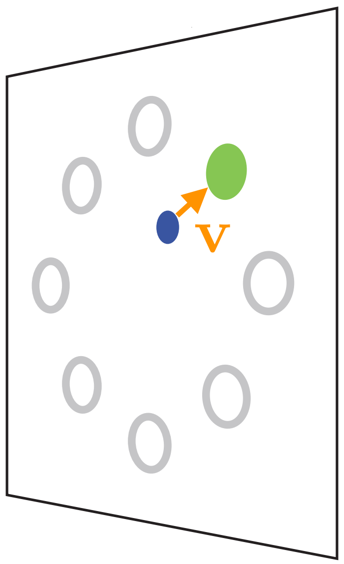

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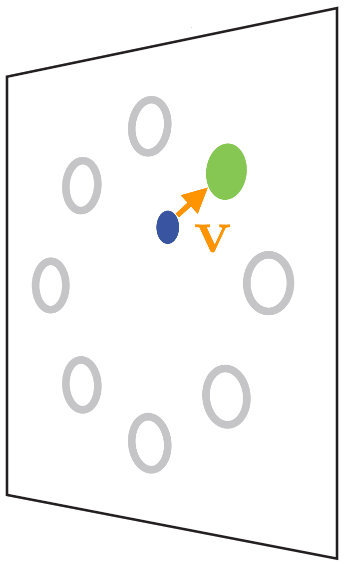

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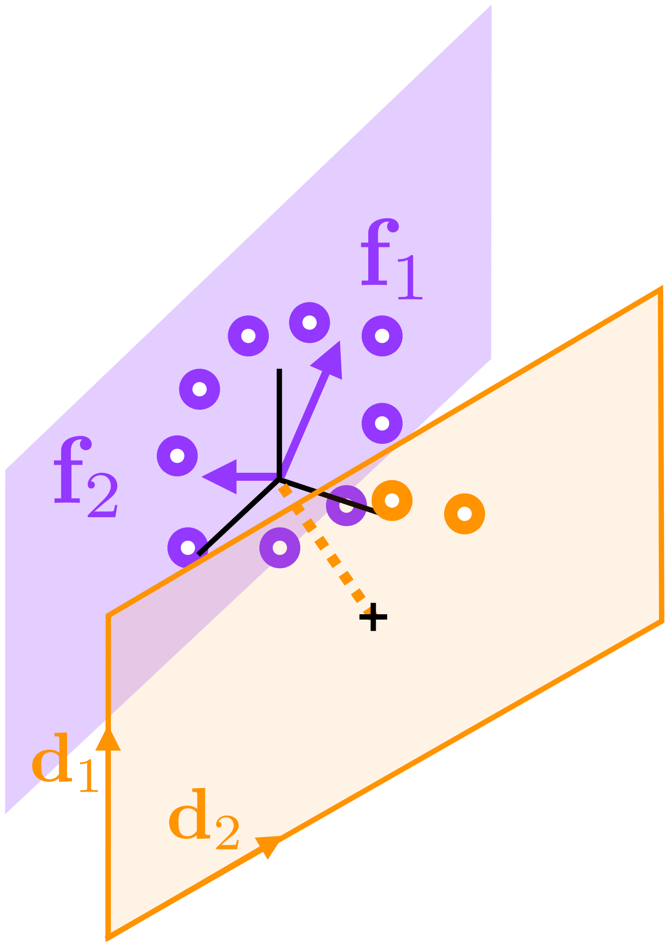

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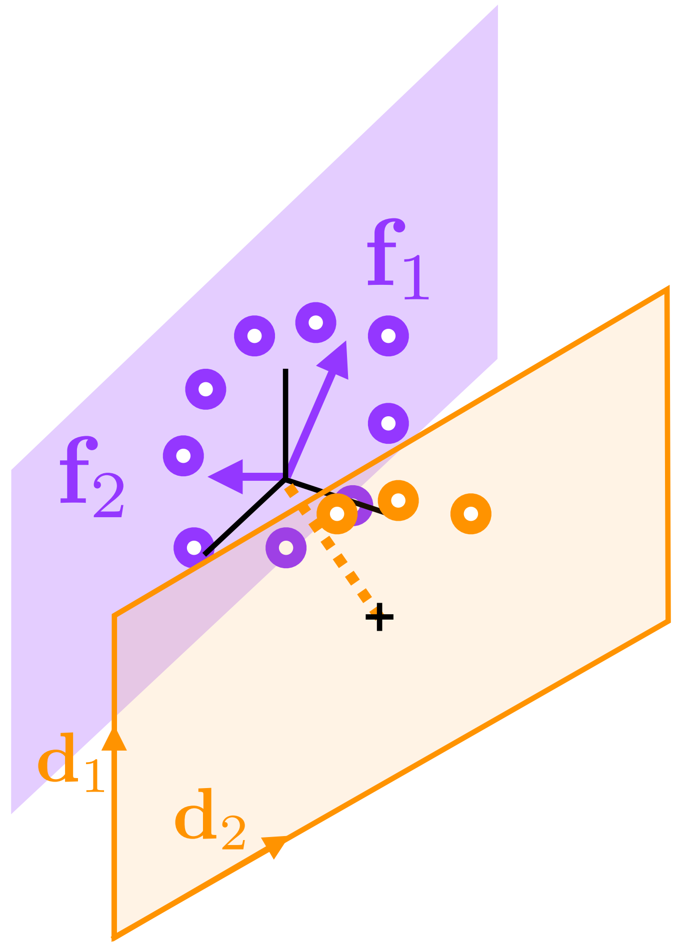

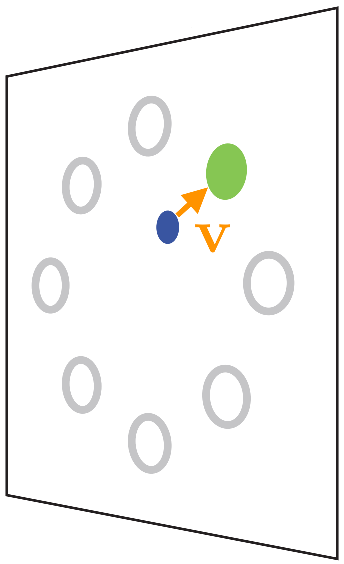

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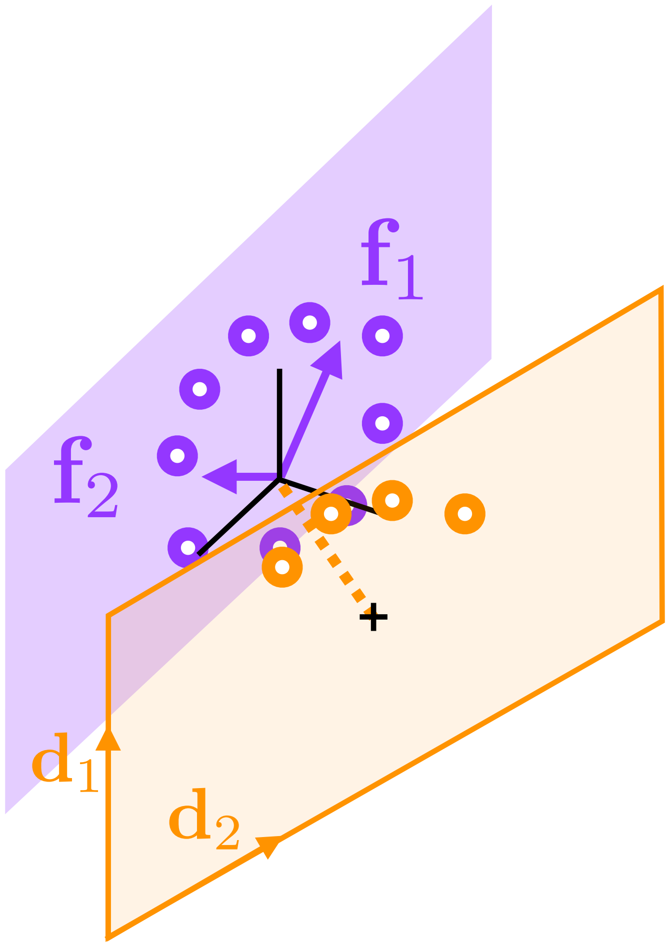

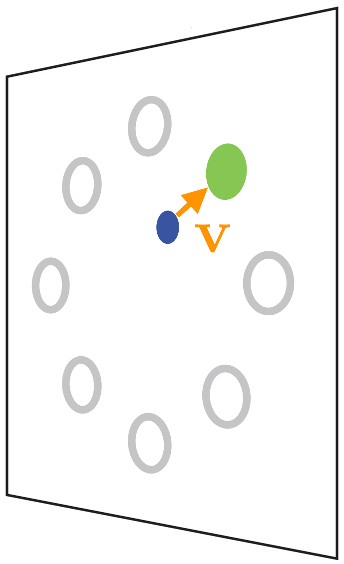

$$\mathbf{v}(t) = \mathbf{D}\mathbf{x}_\theta(t)$$

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(1) low-dimensional activity } **"re-aiming"**

Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

$$\tau\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}^{rec}\mathbf{x} + \mathbf{W}^{in}\left(\mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2\right)$$

[θ constant]

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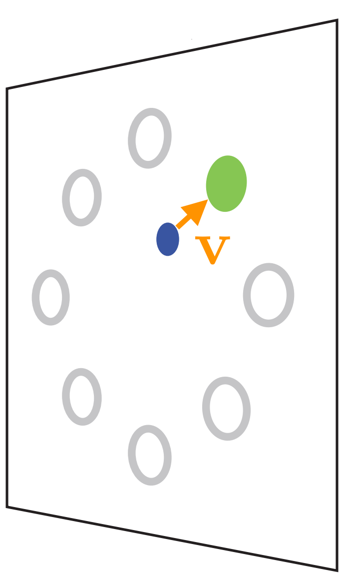

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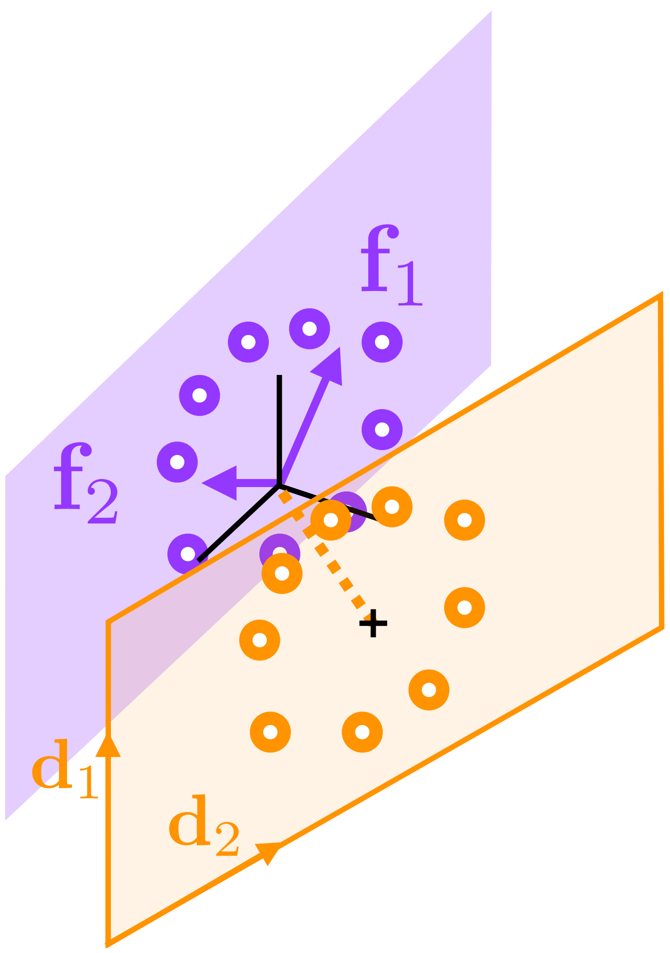

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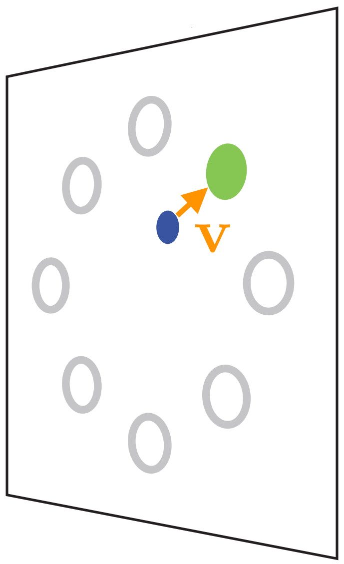

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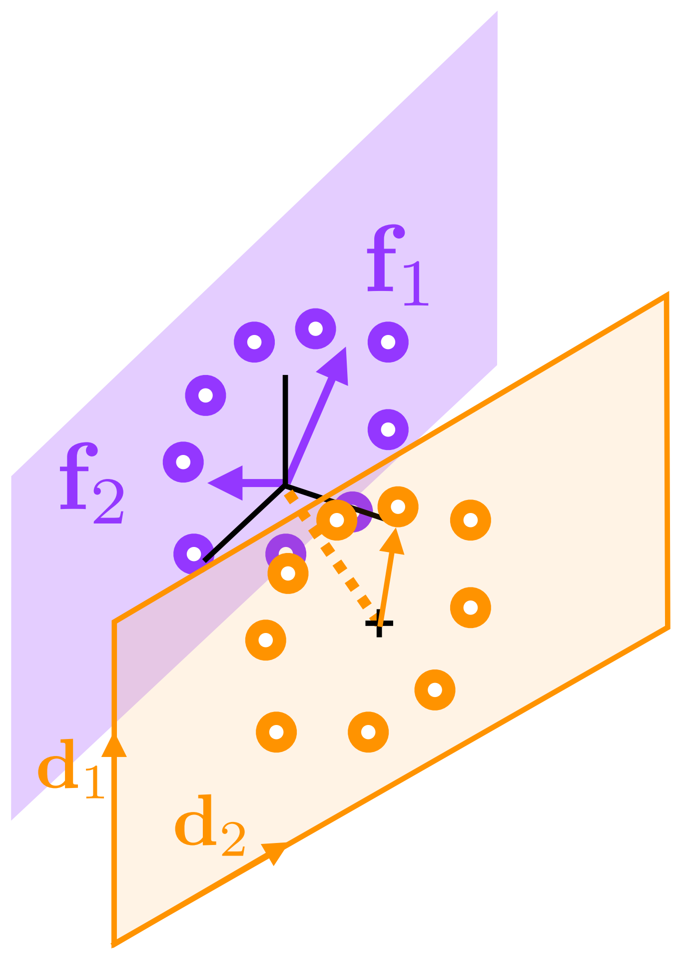

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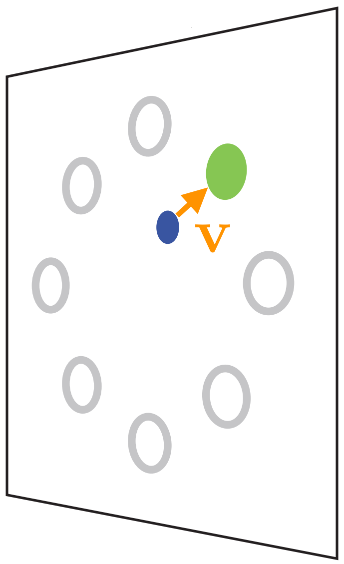

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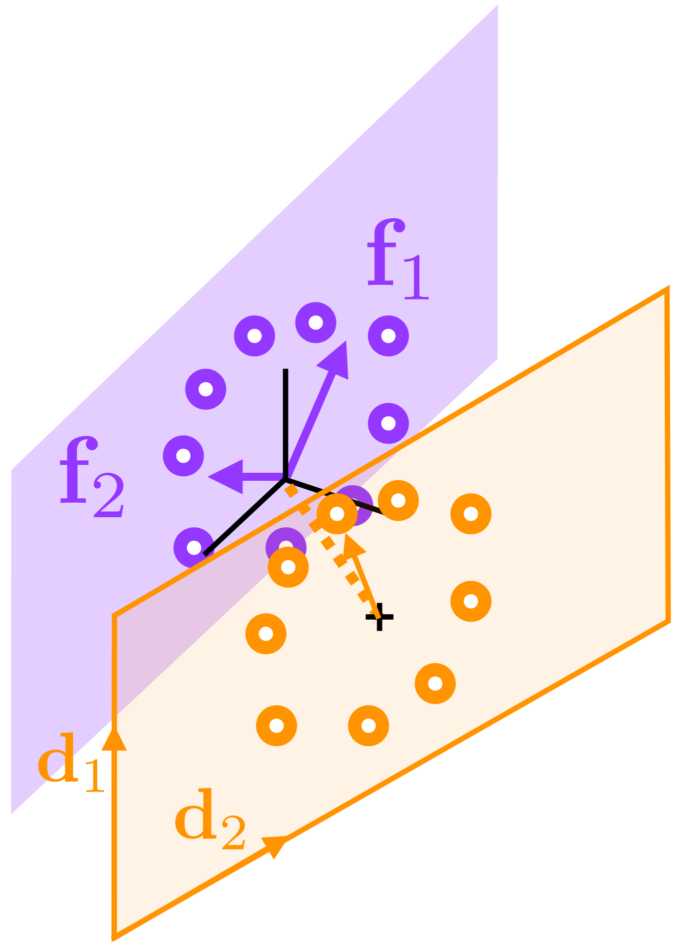

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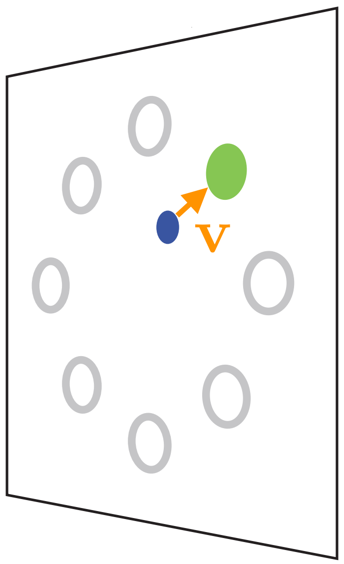

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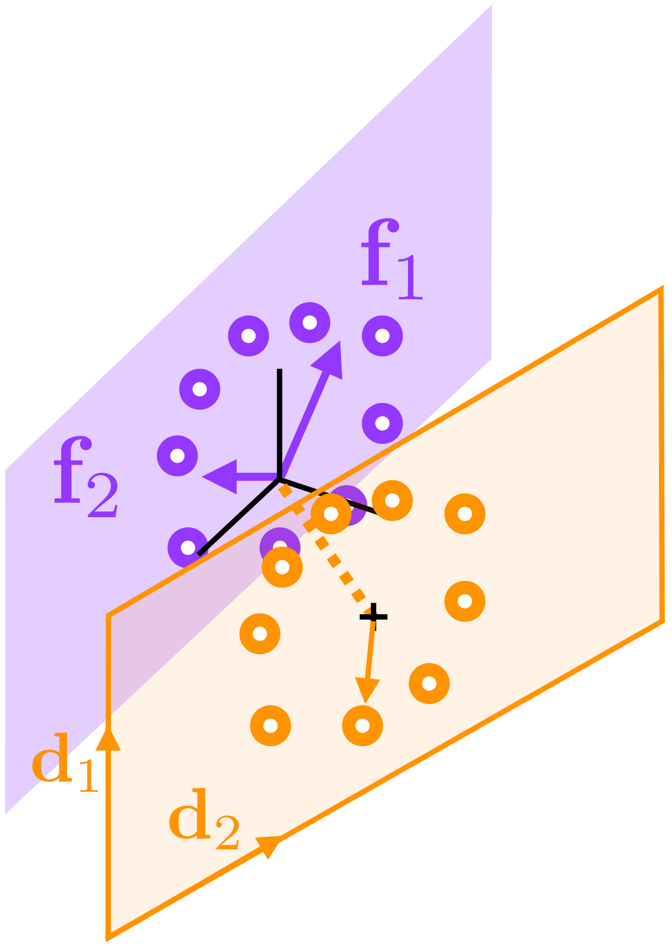

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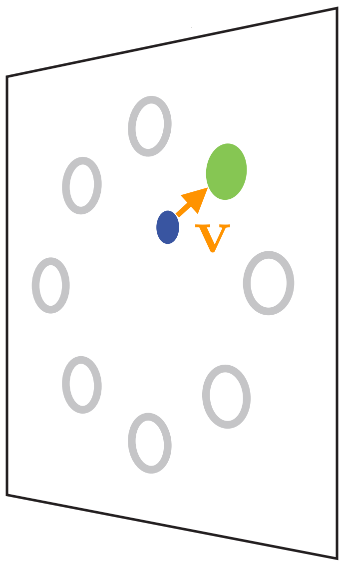

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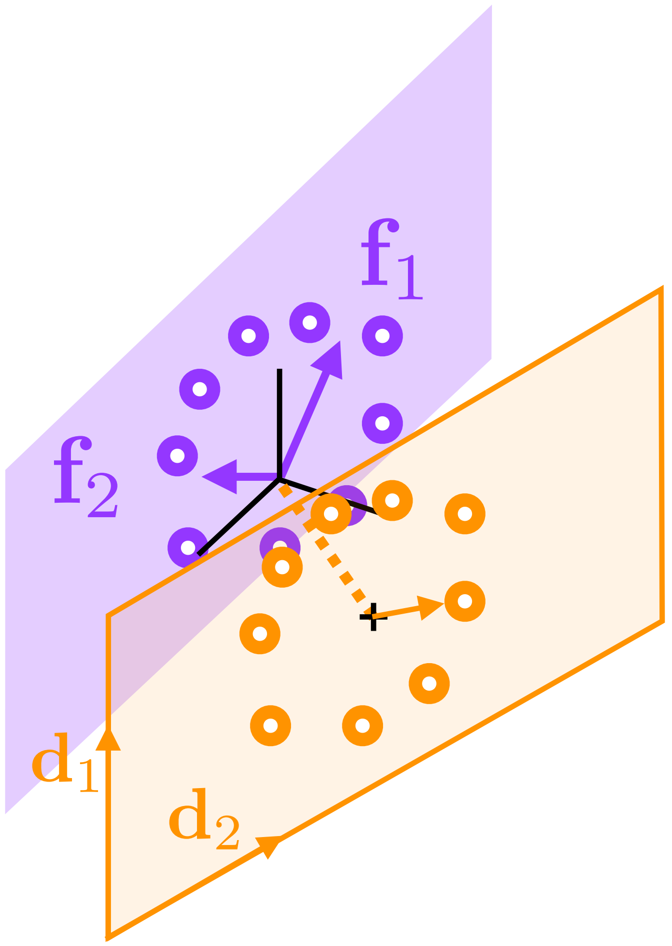

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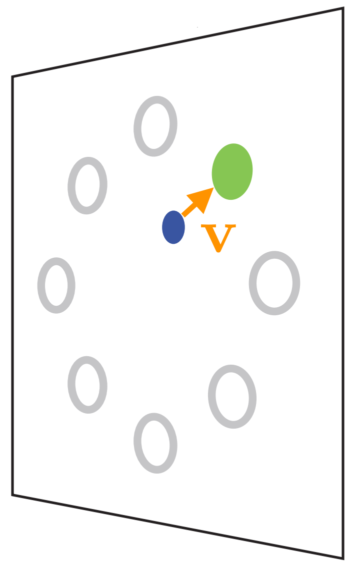

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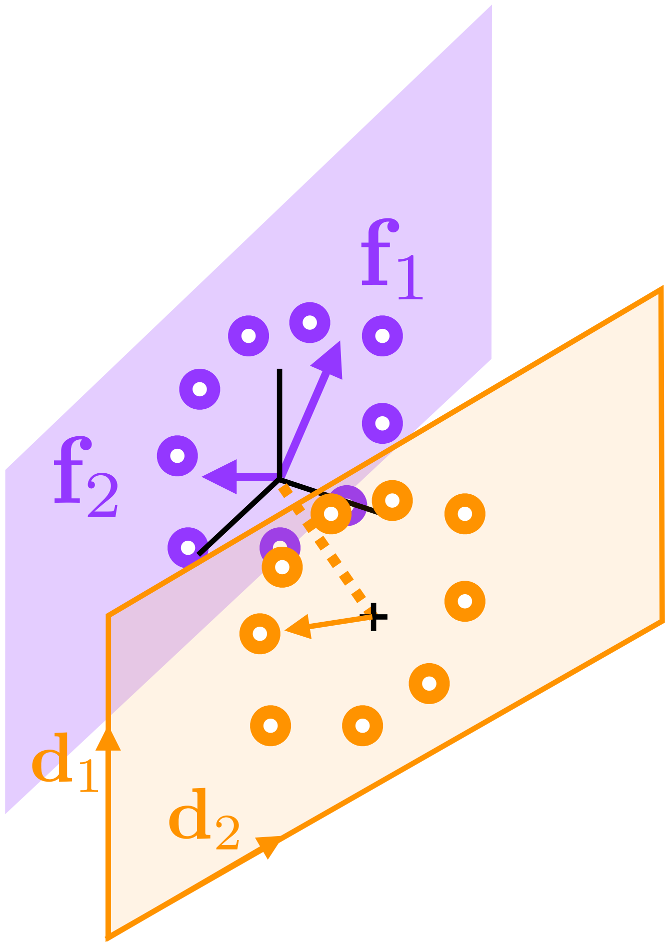

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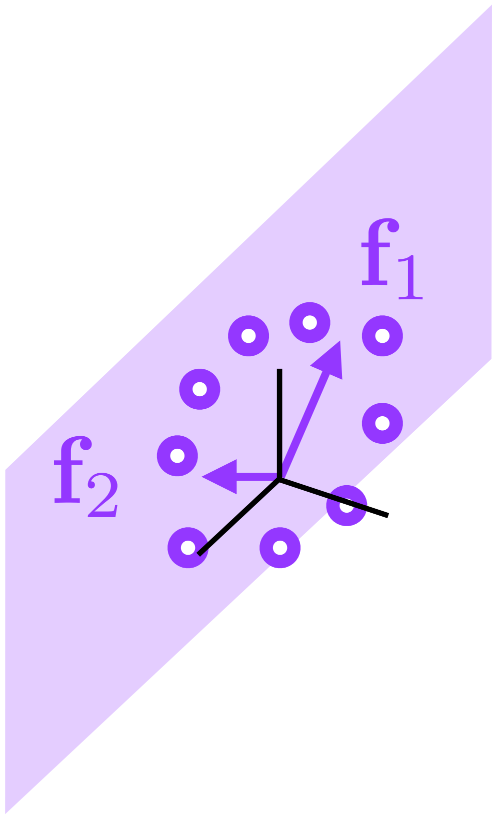

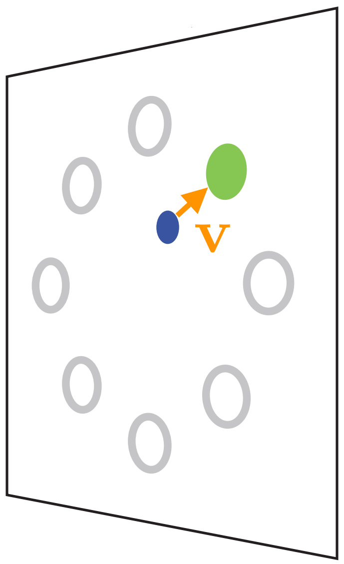

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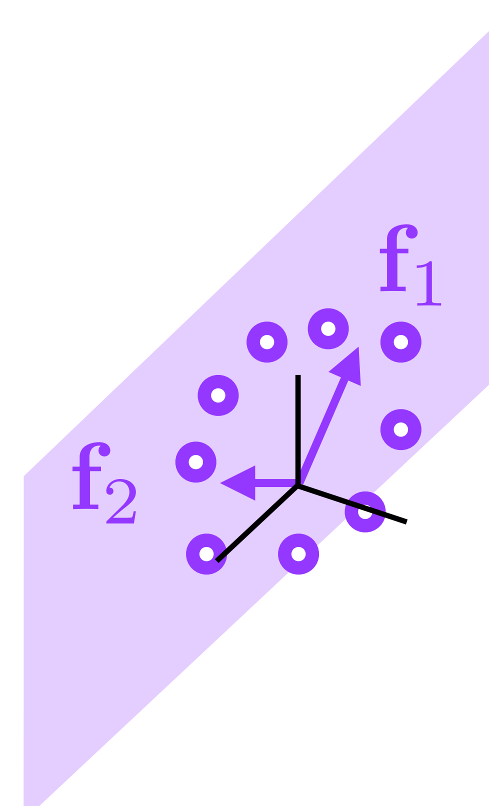

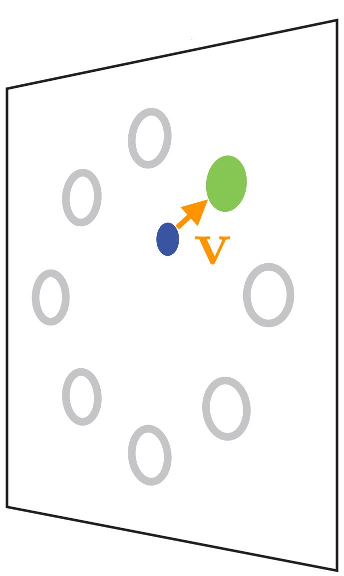

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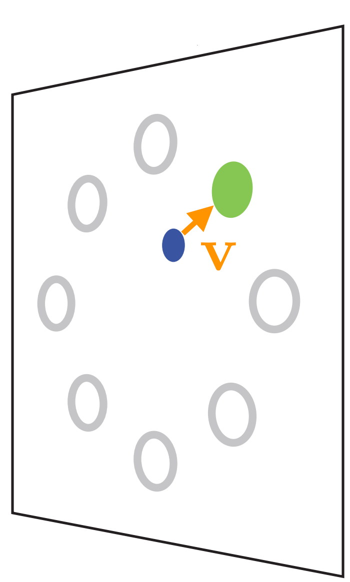

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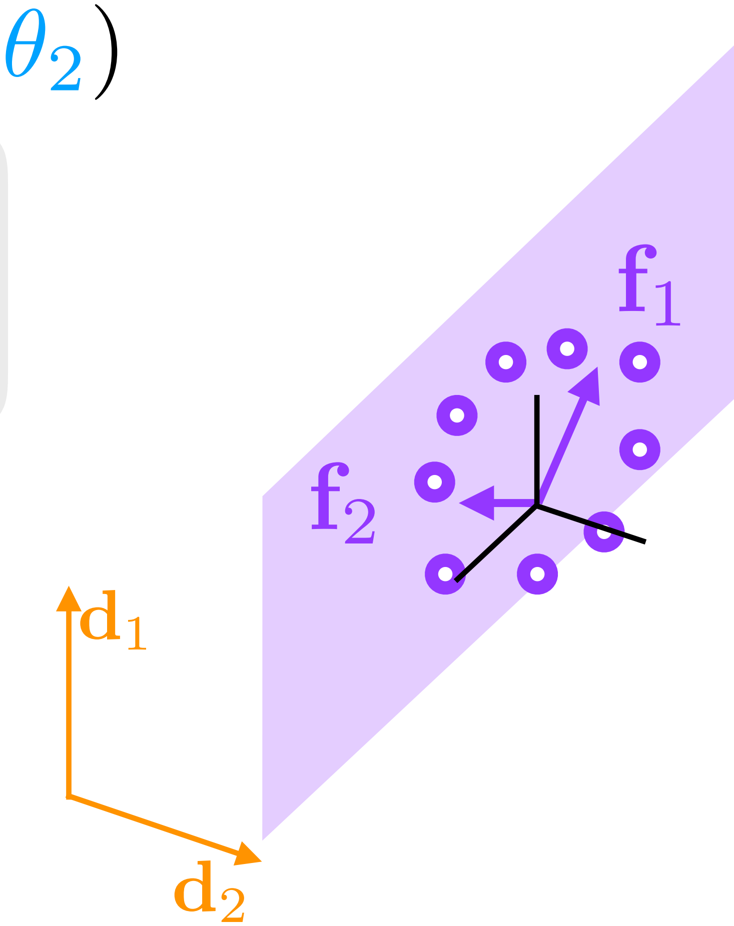

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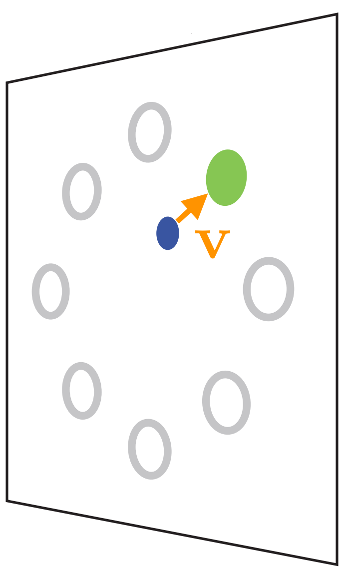

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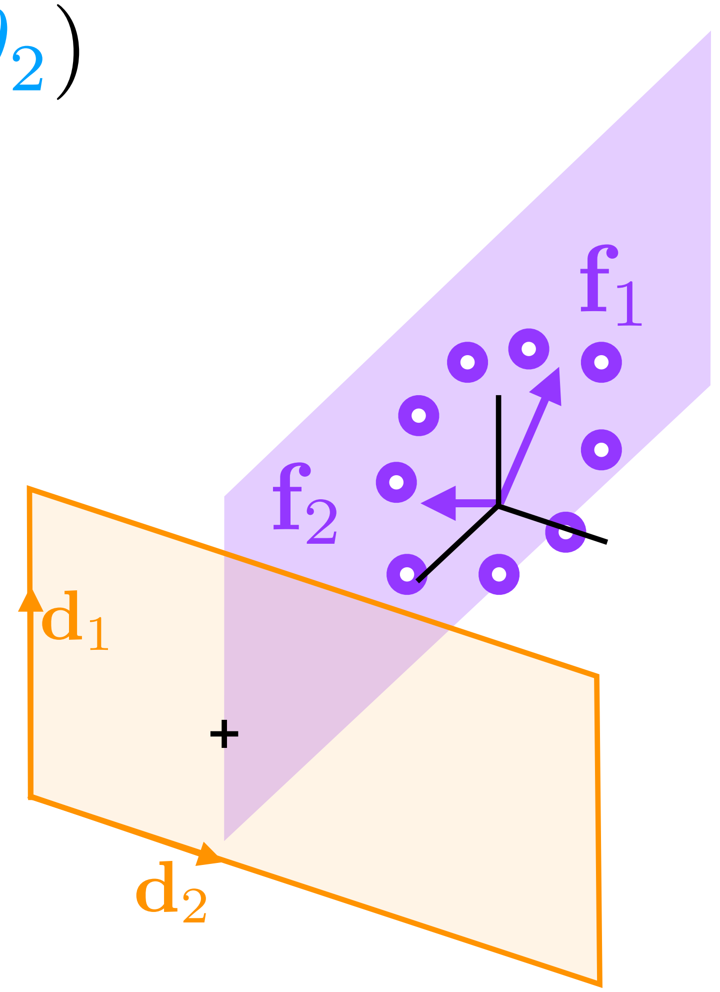

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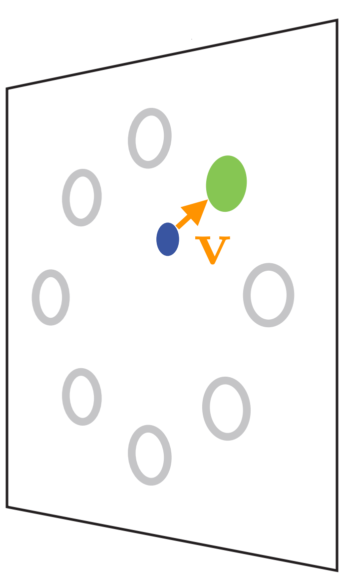

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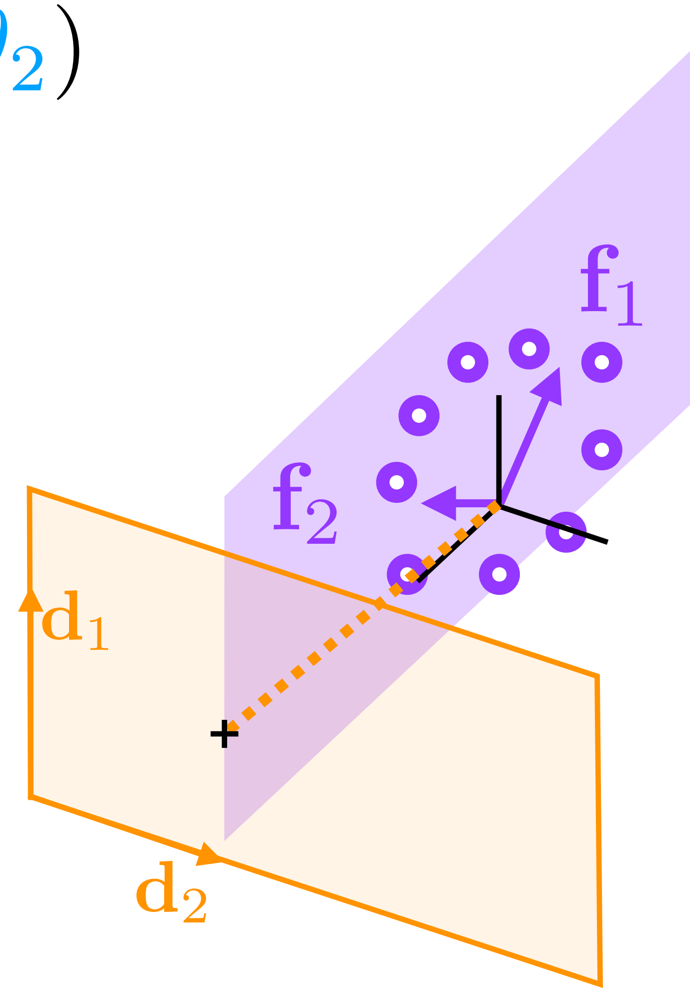

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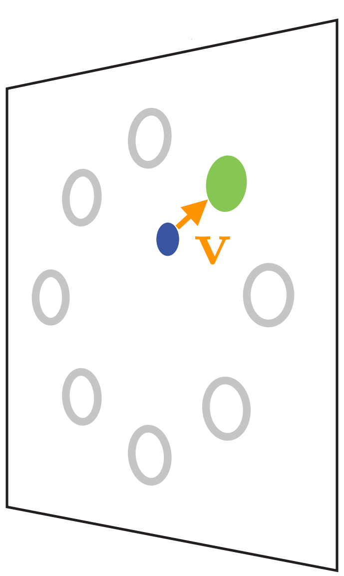

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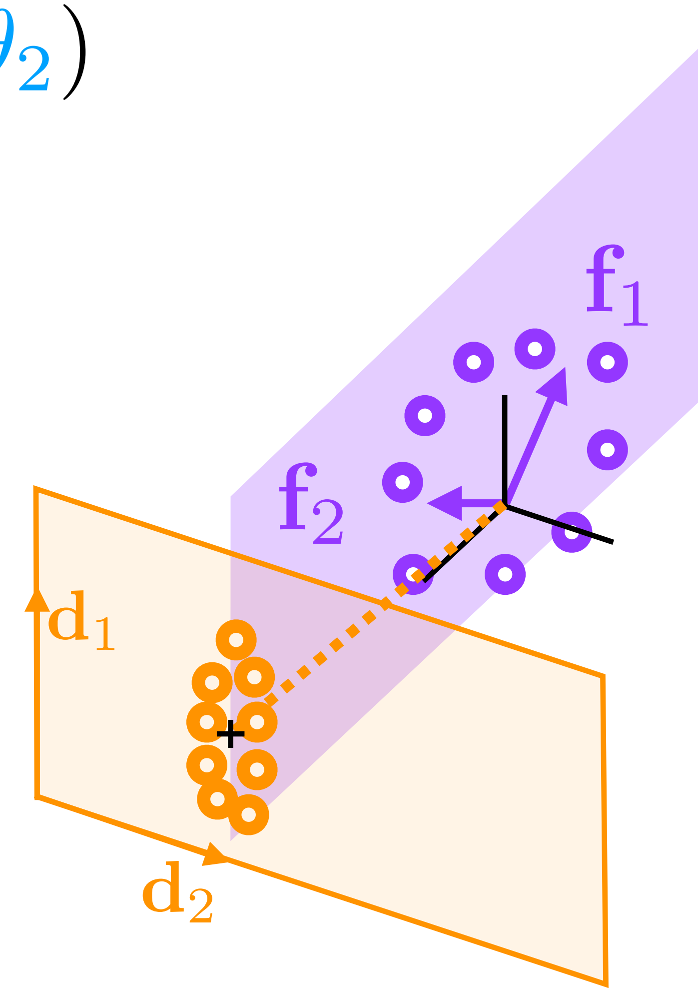

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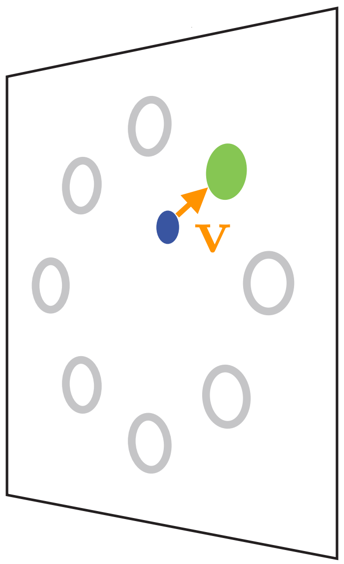

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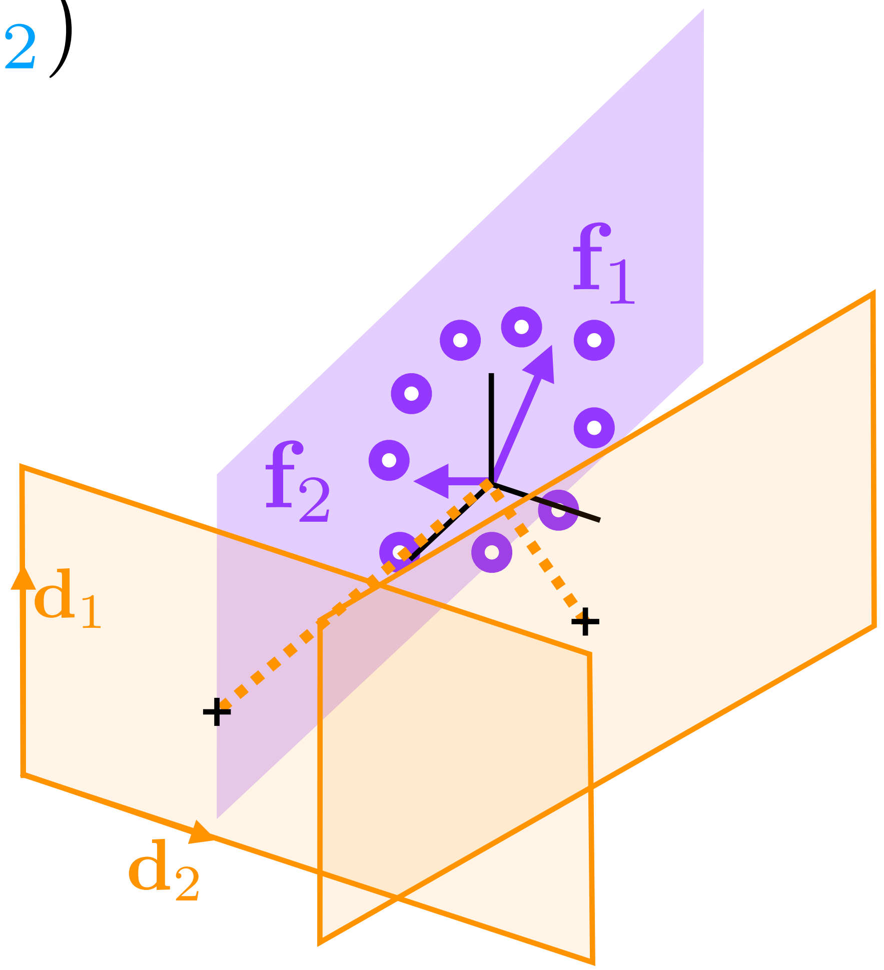

(1) low-dimensional activity

(2) learning ~ alignment

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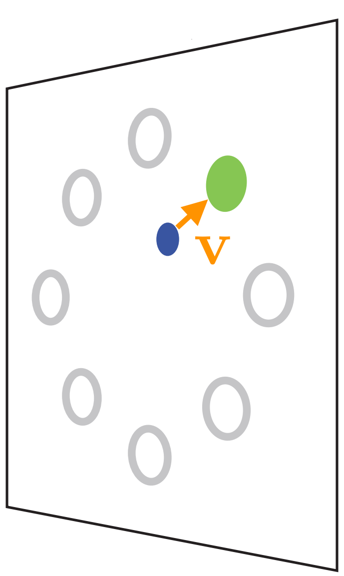

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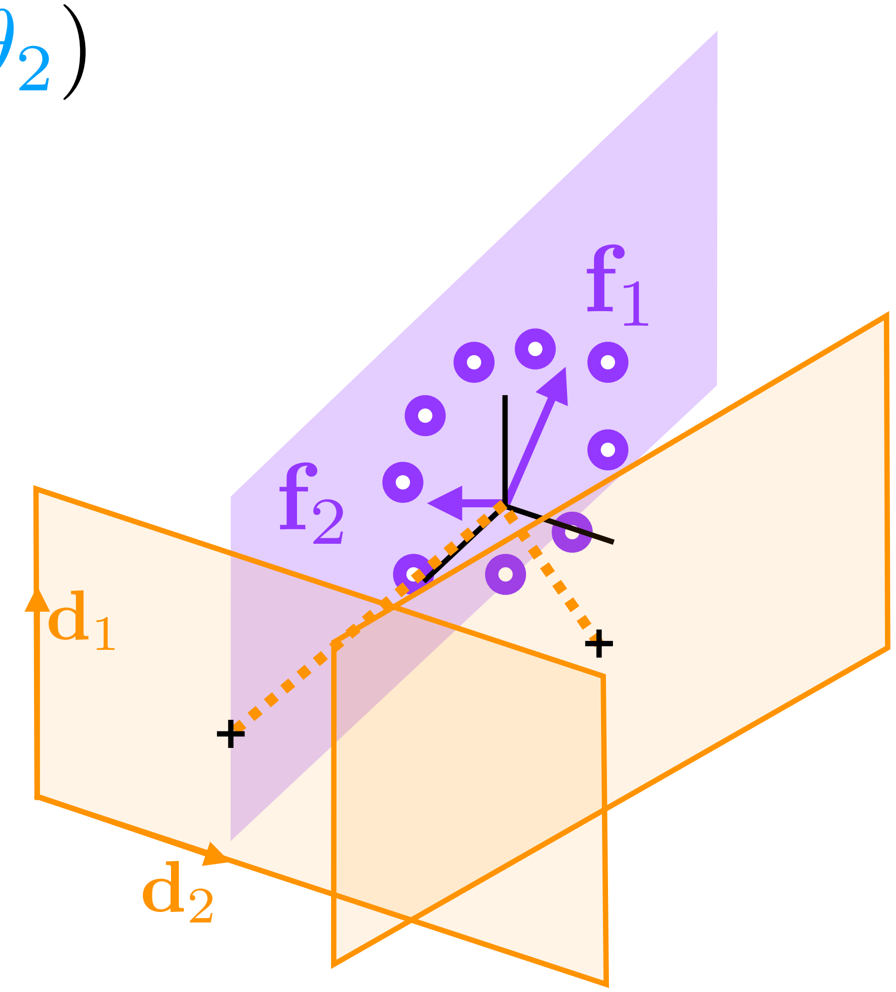

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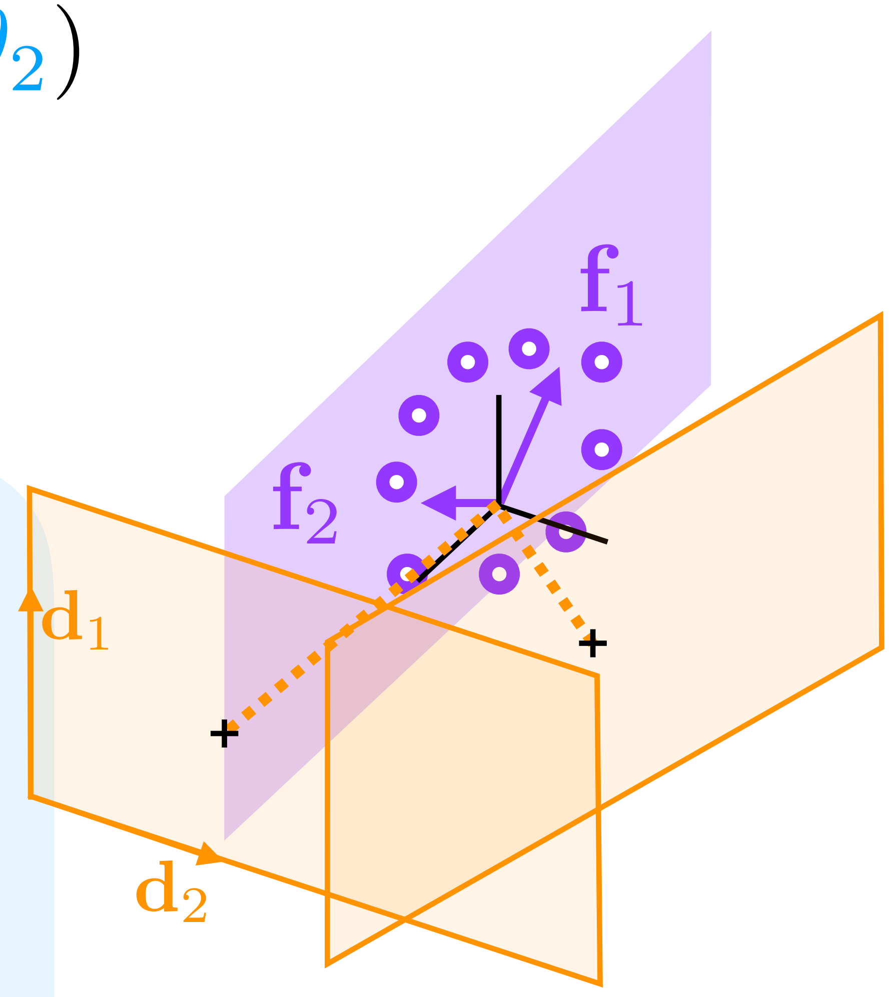
$\mathbf{D}\mathbf{x}_\theta(t) \quad \mathbf{v}^*$



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(2) learning ~ alignment

} **“re-aiming”**

Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

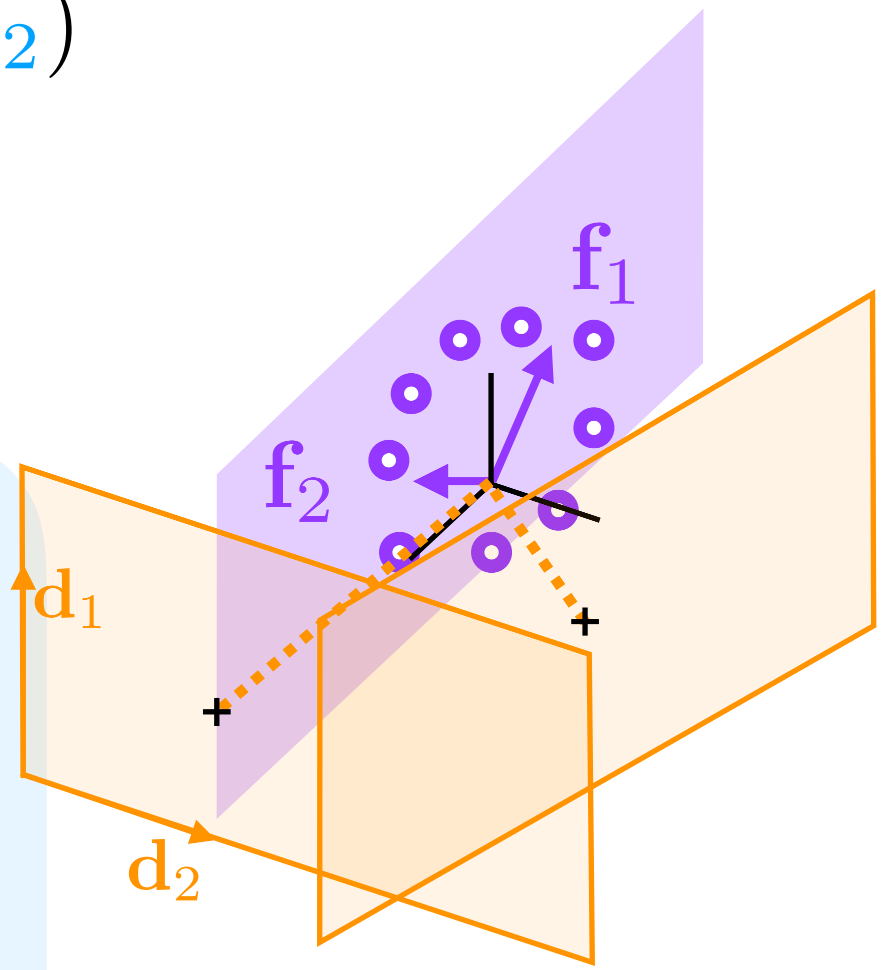
$$\tau\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}^{rec}\mathbf{x} + \mathbf{W}^{in}\left(\mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2\right)$$

[θ constant]

$$\Rightarrow \mathbf{x}_\theta(t) = \mathbf{f}_1\theta_1 + \mathbf{f}_2\theta_2$$

$$\tau\dot{\mathbf{f}}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

$$\|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2$$



(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

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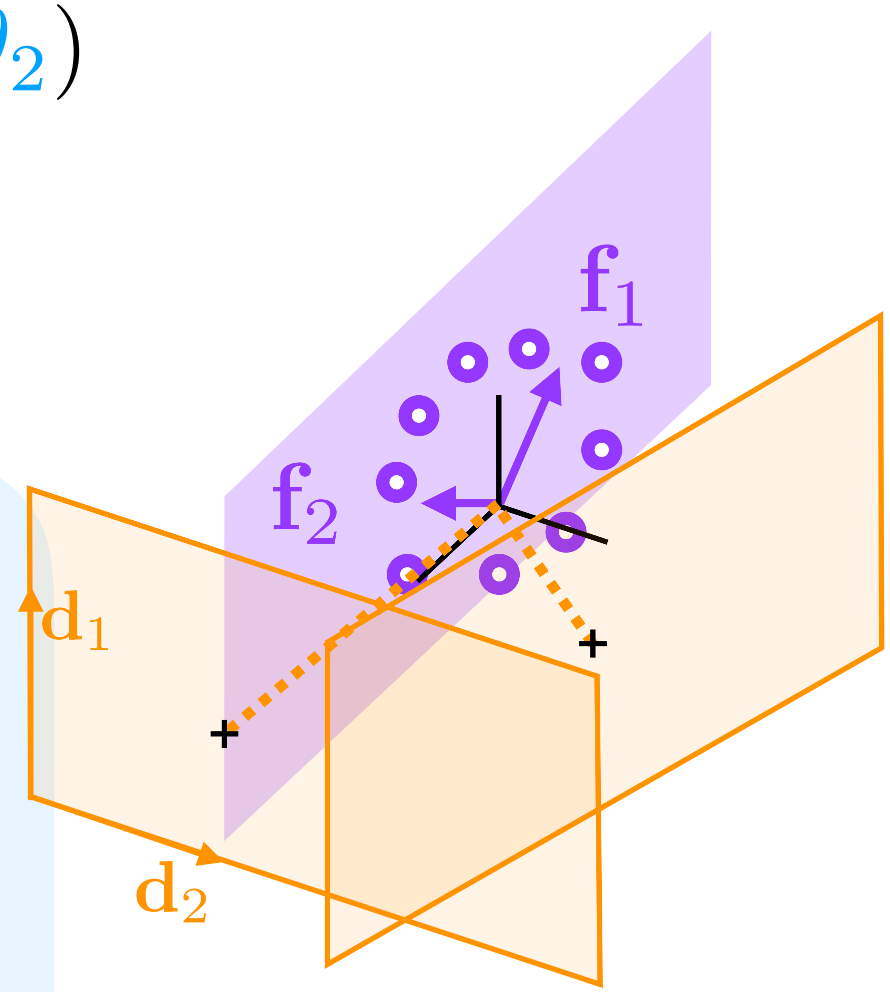
$$\Rightarrow \mathbf{x}_\theta(t) = \mathbf{f}_1\theta_1 + \mathbf{f}_2\theta_2$$

$$\tau\dot{\mathbf{f}}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

$$\min_{\substack{\boldsymbol{\theta} \\ \|\mathbf{u}_\theta\|<C}} \quad \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2$$



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(2) learning ~ alignment

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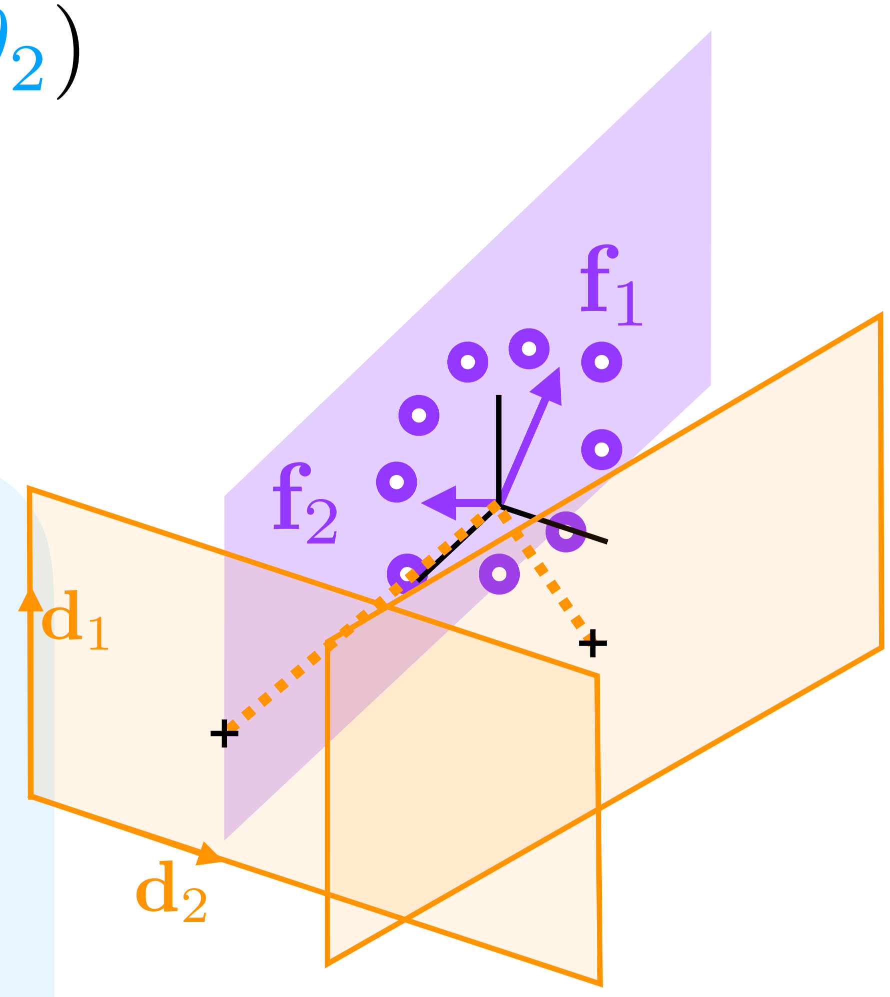
$$\tau\dot{\mathbf{f}}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

$$\mathbb{E}\left[\min_{\substack{\boldsymbol{\theta} \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2\right]$$



(1) low-dimensional activity

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Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

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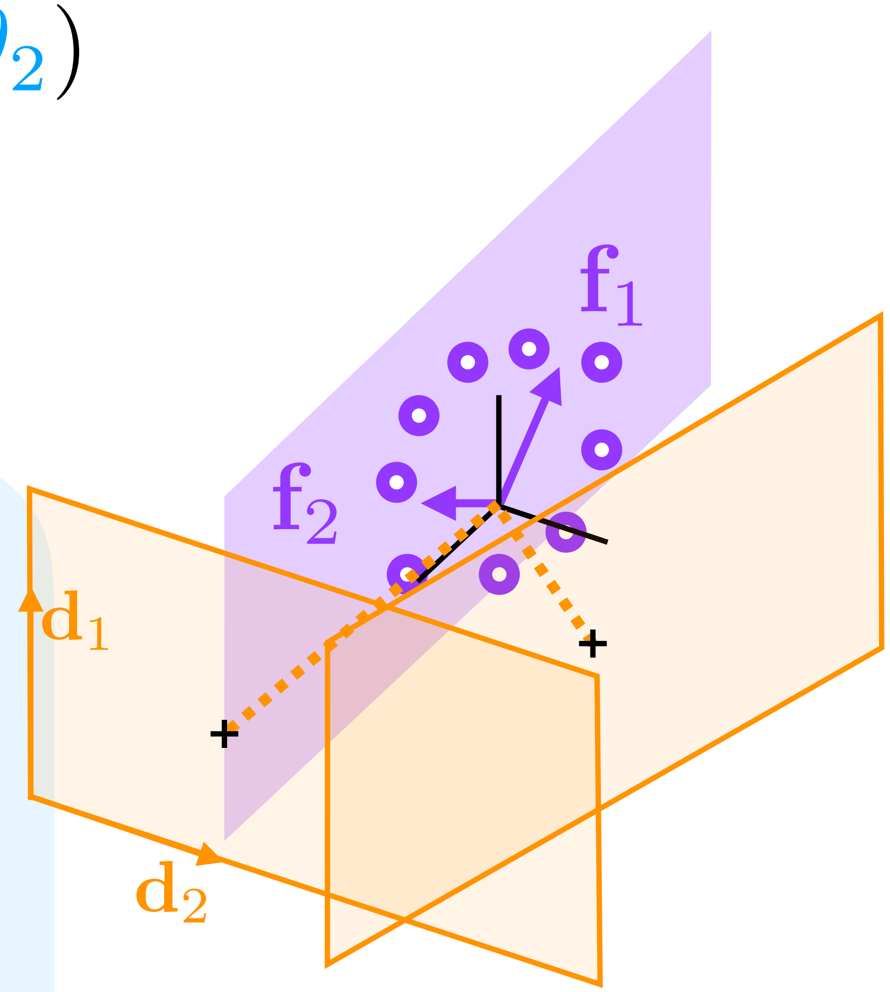
$$\tau\dot{\mathbf{f}}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

optimal avg reaching error

$$\mathbb{E}\left[\min_{\substack{\boldsymbol{\theta} \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2\right]$$



(1) low-dimensional activity

(2) learning ~ alignment

} **"re-aiming"**

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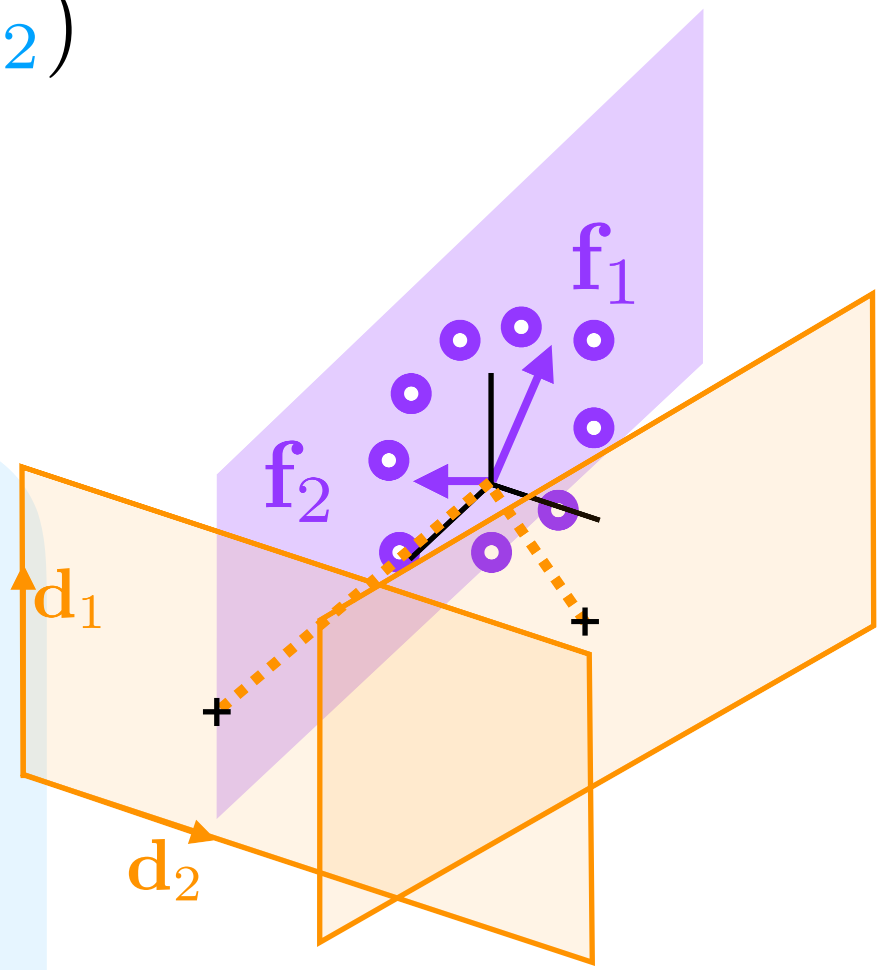
optimal avg reaching error

$$\mathbb{E}\left[\min_{\substack{\boldsymbol{\theta} \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2\right]$$

$$= \frac{1}{2}\sum_{i=1}^{2}\frac{1}{\left(\gamma {s_i}^2 + 1\right)^2}$$

decoder alignment



(1) low-dimensional activity

(2) learning ~ alignment

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Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

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[θ constant]

$$\Rightarrow \mathbf{x}_\theta(t) = \mathbf{f}_1\theta_1 + \mathbf{f}_2\theta_2$$

$$\tau\dot{\mathbf{f}}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

optimal avg reaching error

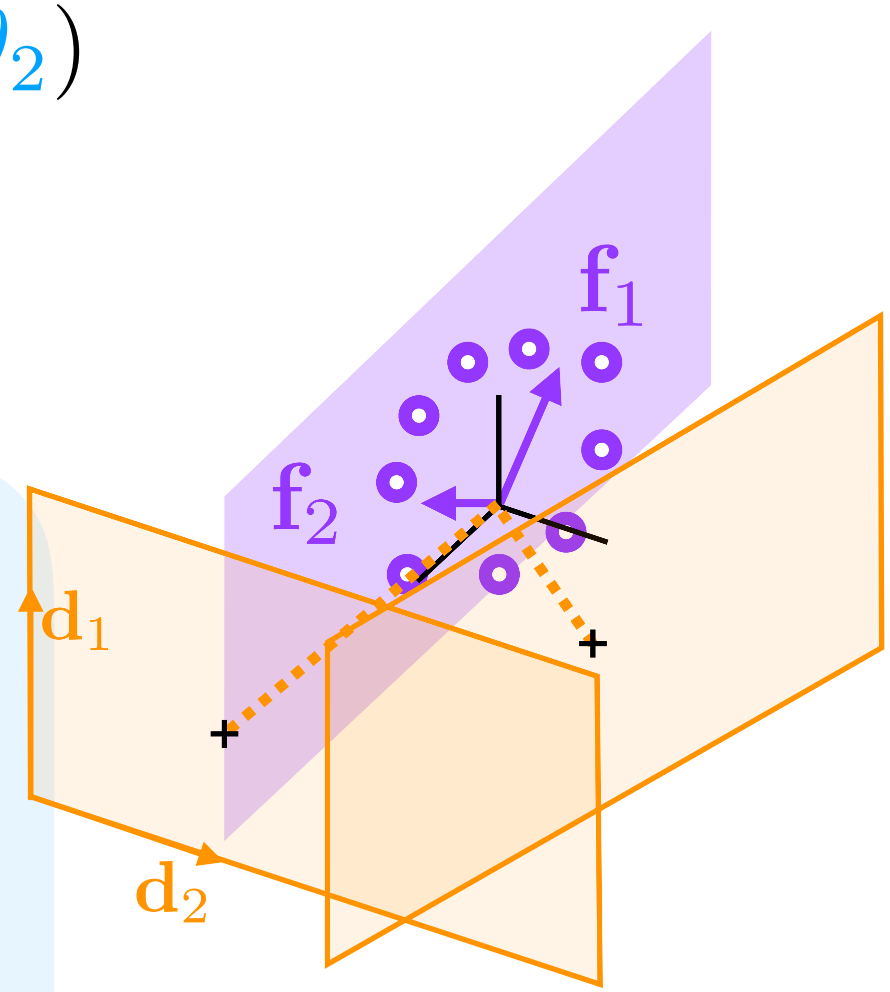
$$\mathbb{E}\left[\min_{\substack{\boldsymbol{\theta} \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2\right]$$

s_i = singular values of

$$\begin{bmatrix} \mathbf{d}_1^T\mathbf{f}_1 & \mathbf{d}_1^T\mathbf{f}_2 \\ \mathbf{d}_2^T\mathbf{f}_1 & \mathbf{d}_2^T\mathbf{f}_2 \end{bmatrix}$$

$$= \frac{1}{2}\sum_{i=1}^{2}\frac{1}{\left(\gamma {s_i}^2 + 1\right)^2}$$

decoder alignment



(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

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[θ constant]

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$$\tau\dot{\mathbf{f}}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

optimal avg reaching error

$$\mathbb{E}\left[\min_{\substack{\boldsymbol{\theta} \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2\right] = \frac{1}{2}\sum_{i=1}^{2}\frac{1}{\left(\gamma {s_i}^2 + 1\right)^2}$$

s_i = singular values of

$$\begin{bmatrix} \mathbf{d}_1^T\mathbf{f}_1 & \mathbf{d}_1^T\mathbf{f}_2 \\ \mathbf{d}_2^T\mathbf{f}_1 & \mathbf{d}_2^T\mathbf{f}_2 \end{bmatrix}$$

decoder alignment

(1) low-dimensional activity

(2) learning ~ alignment

} **“re-aiming”**

Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

$$\tau\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}^{rec}\mathbf{x} + \mathbf{W}^{in}\left(\mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2\right)$$

$\ldots + \mathbf{f}_2\theta_2$

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$$\tau\mathbf{f}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

optimal avg reaching error

$$\mathbb{E}\left[\min_{\substack{\theta \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2\right]$$

s_i = singular values of

$$\begin{bmatrix} \mathbf{d}_1^T\mathbf{f}_1 & \mathbf{d}_1^T\mathbf{f}_2 \\ \mathbf{d}_2^T\mathbf{f}_1 & \mathbf{d}_2^T\mathbf{f}_2 \end{bmatrix} \quad = \frac{1}{2}\sum_{i=1}^{2}\frac{1}{(\gamma s_i^2 + 1)^2}$$

decoder alignment

(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

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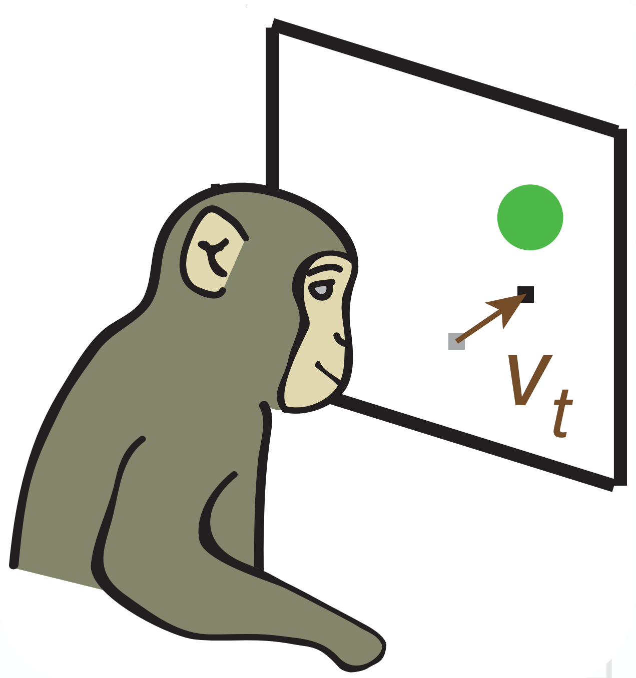

(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2$

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$$\tau\mathbf{f}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

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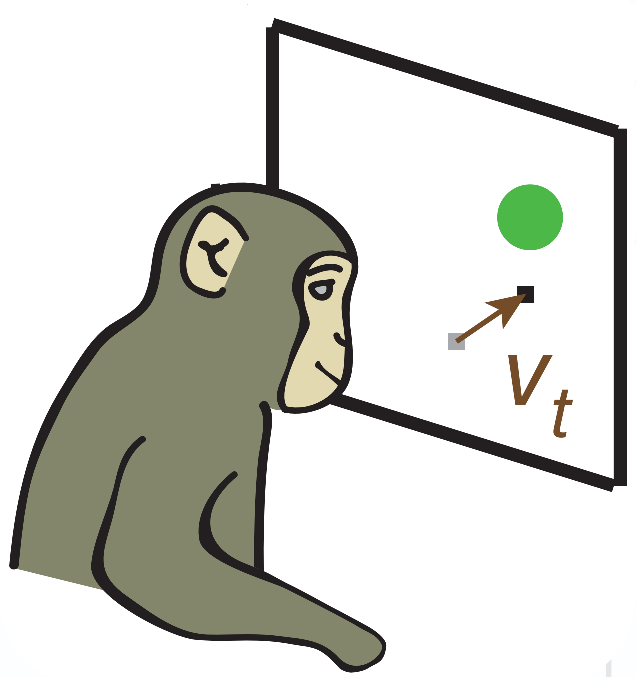
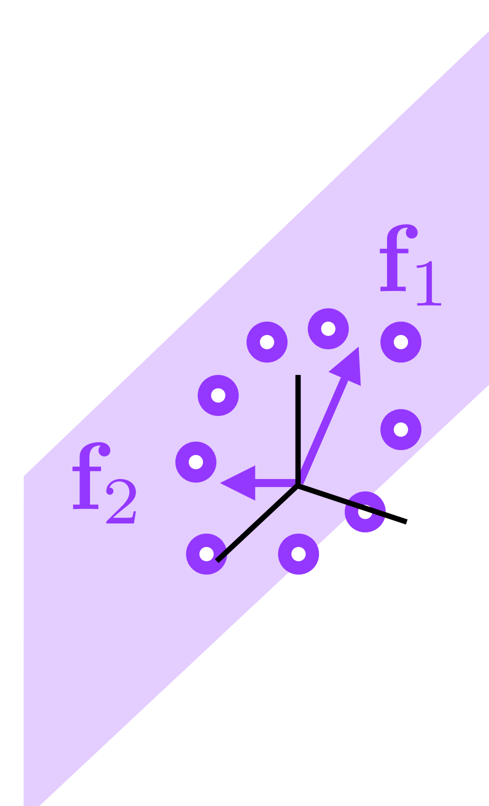


$$\min_{\substack{\theta \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2$$

$$= \frac{1}{2}\sum_{i=1}^{2}\underbrace{\frac{1}{\left(\gamma s_i{}^2 + 1\right)^2}}_{\text{decoder alignment}}$$



(1) low-dimensional activity

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$$\tau\dot{\mathbf{f}}_i = -\mathbf{f}_i + \mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$$

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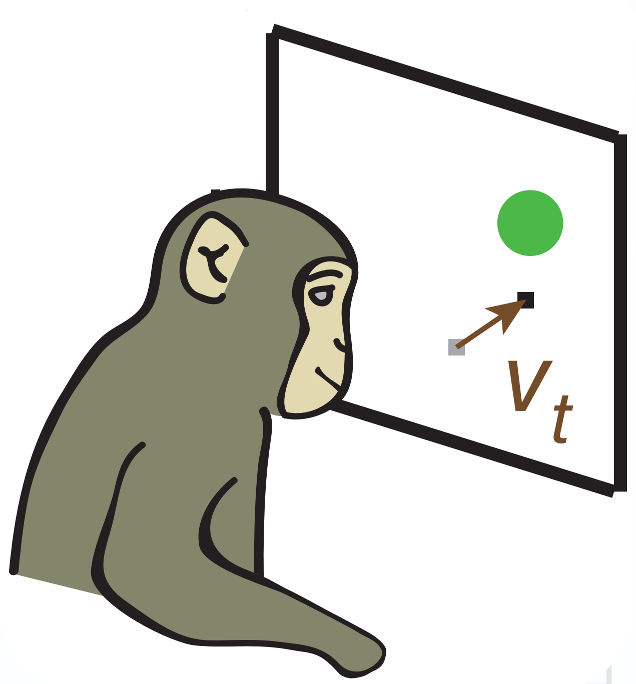
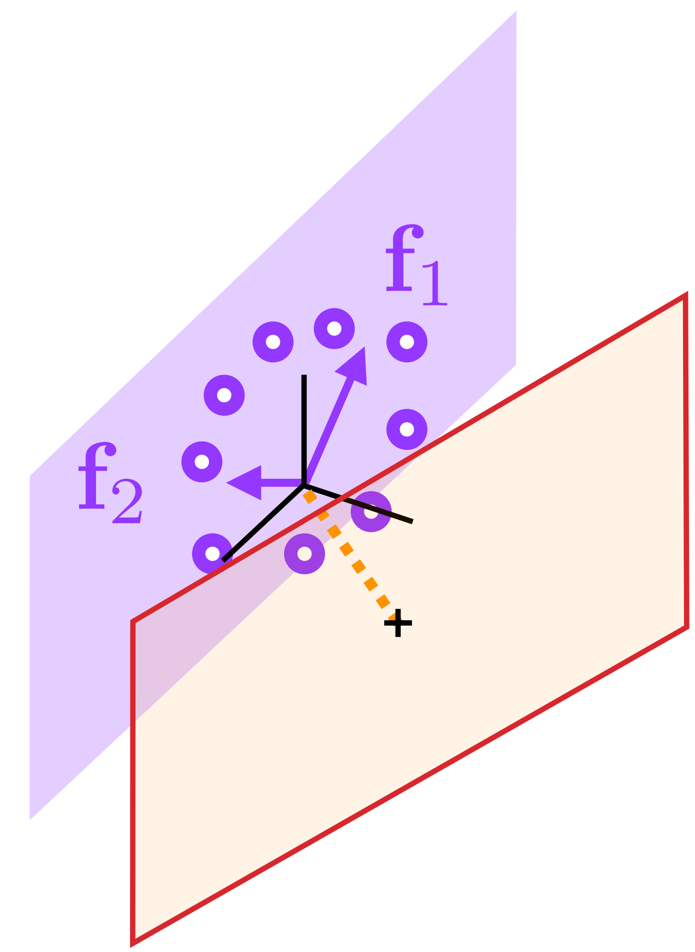


$$\min_{\substack{\boldsymbol{\theta} \\ \|\mathbf{u}_\theta\|<C}} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2$$

$$= \frac{1}{2}\sum_{i=1}^{2} \frac{1}{(\gamma s_i{}^2 + 1)^2}$$

decoder alignment



(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

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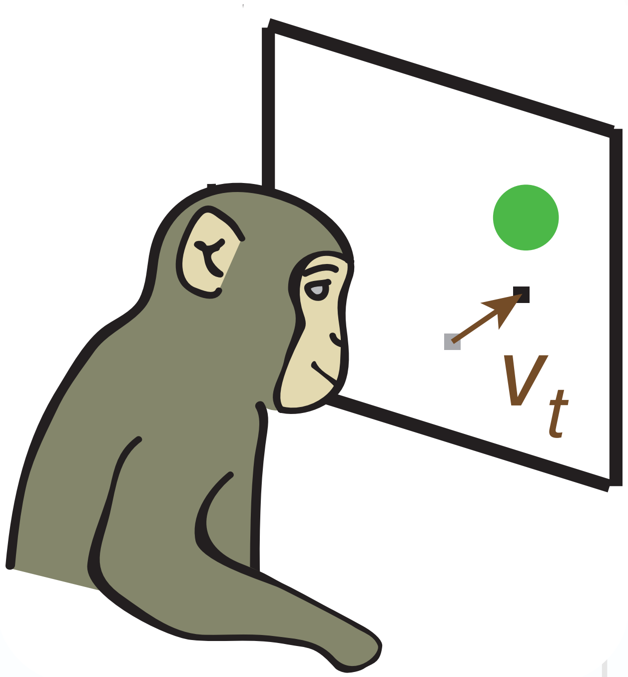

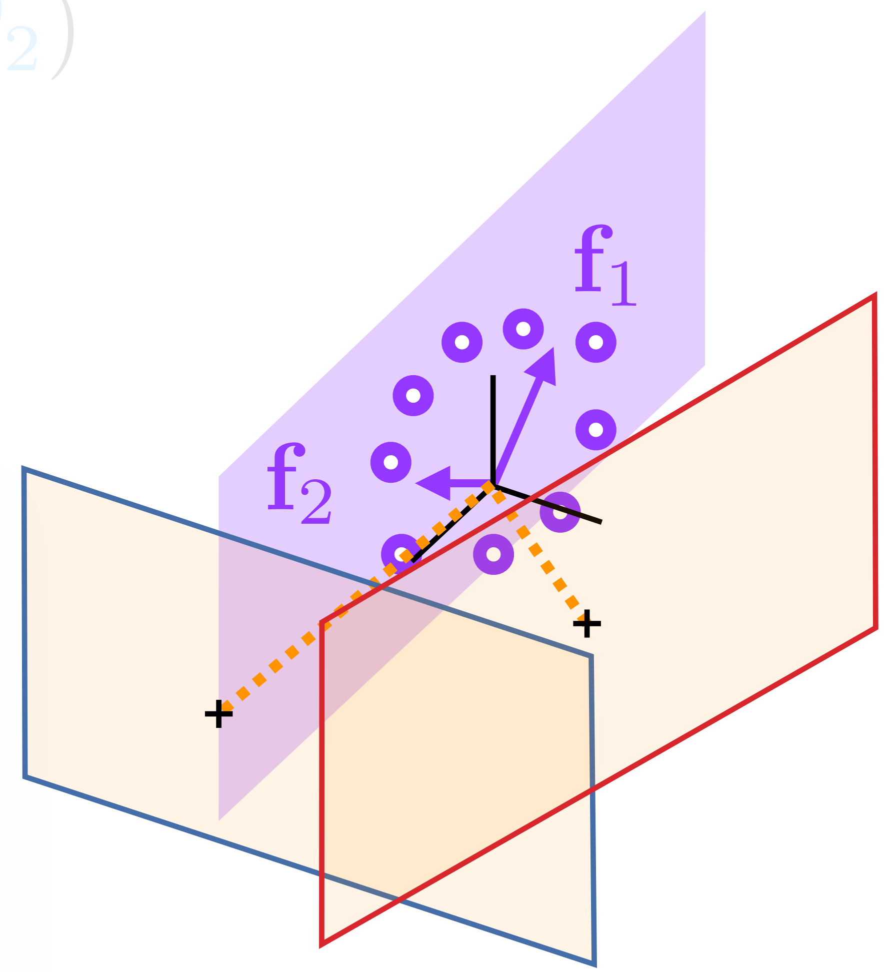

(1) low-dimensional activity

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$$\tau\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}^{rec}\mathbf{x} + \mathbf{W}^{in}(\mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2)$$

$\mathbf{f}_2\theta_2$

$\mathbf{W}^{rec}\mathbf{f}_i + \mathbf{W}^{in}\mathbf{m}_i$

$\min_{\theta} \|\mathbf{D}\mathbf{x}_\theta(t) - \mathbf{v}^*\|^2$

$$= \frac{1}{2}\sum_{i=1}^{2} \frac{1}{(\gamma s_i^2 + 1)^2}$$

decoder alignment

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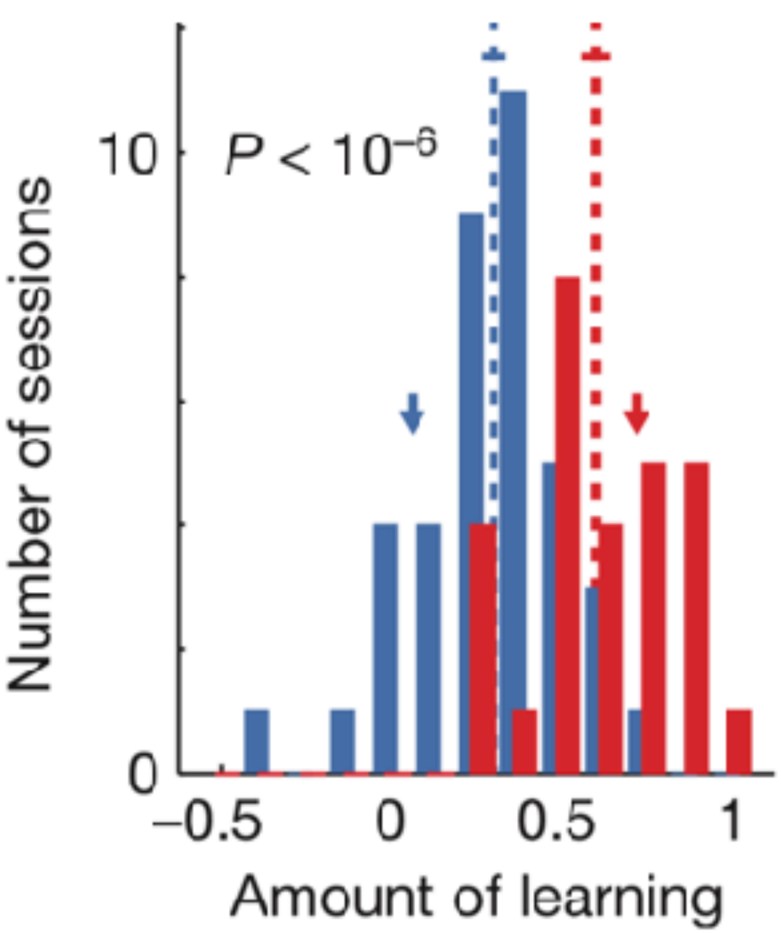

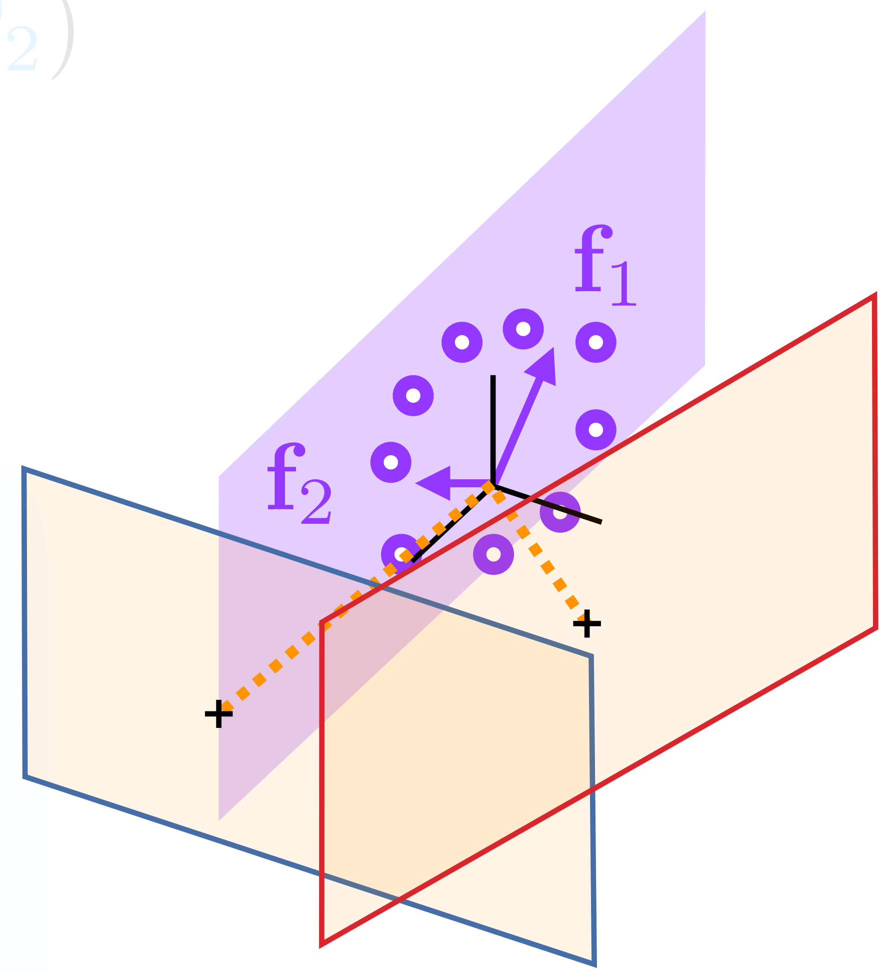

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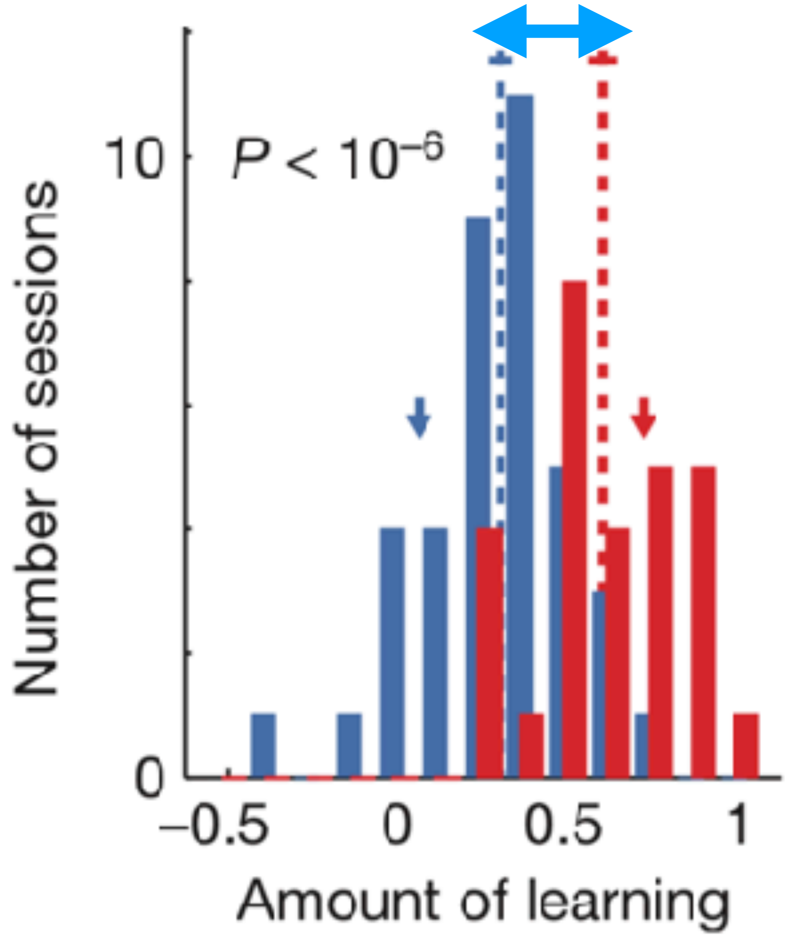
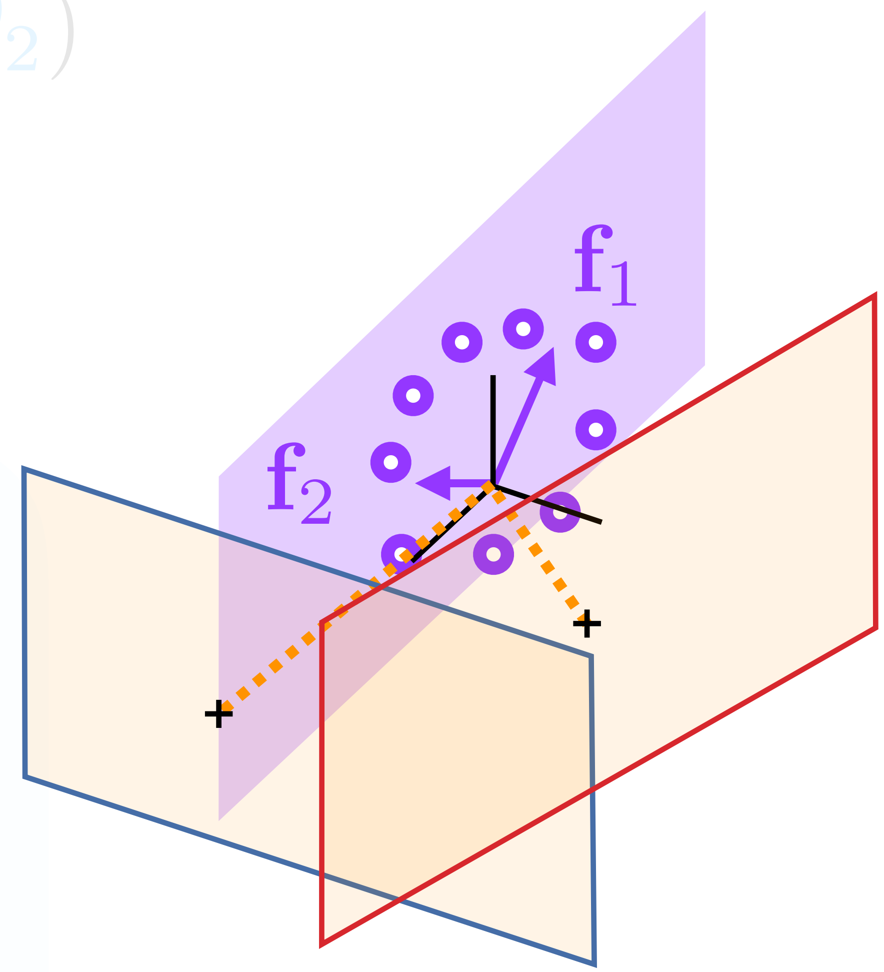
$$= \frac{1}{2}\sum_{i=1}^{2}\frac{1}{\left(\gamma s_i^{\,2} + 1\right)^2}$$

decoder alignment

Sadtler et al. '14





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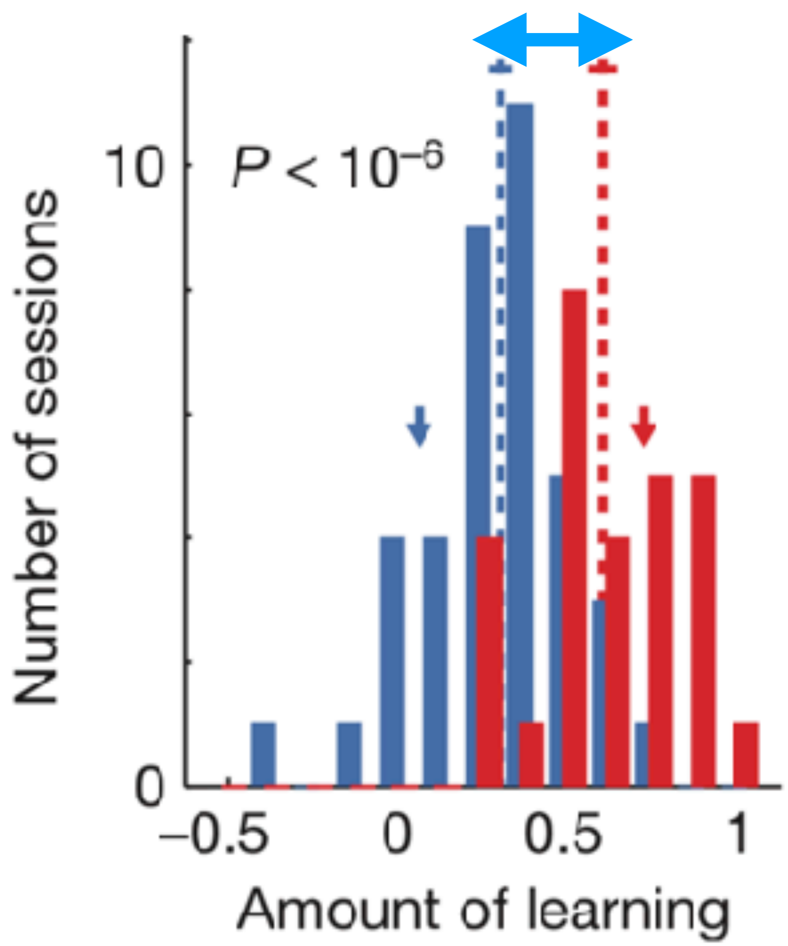

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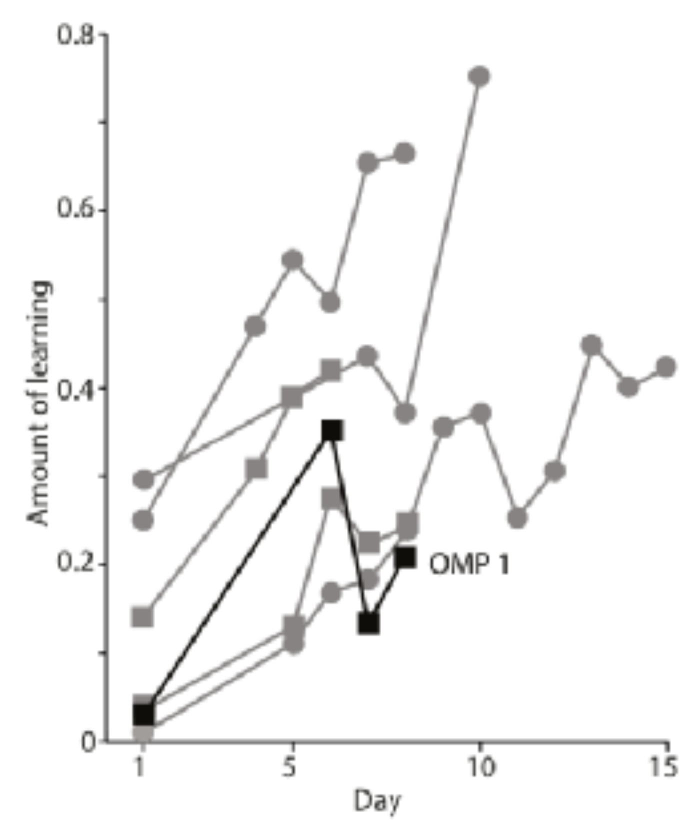

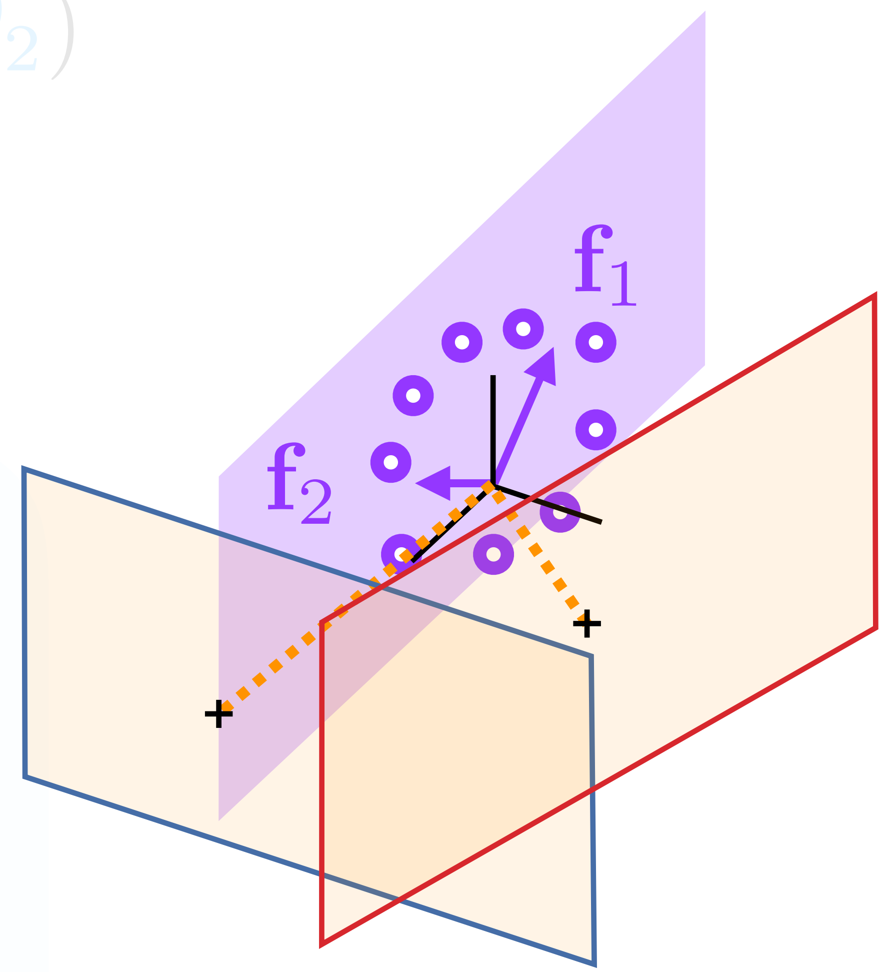

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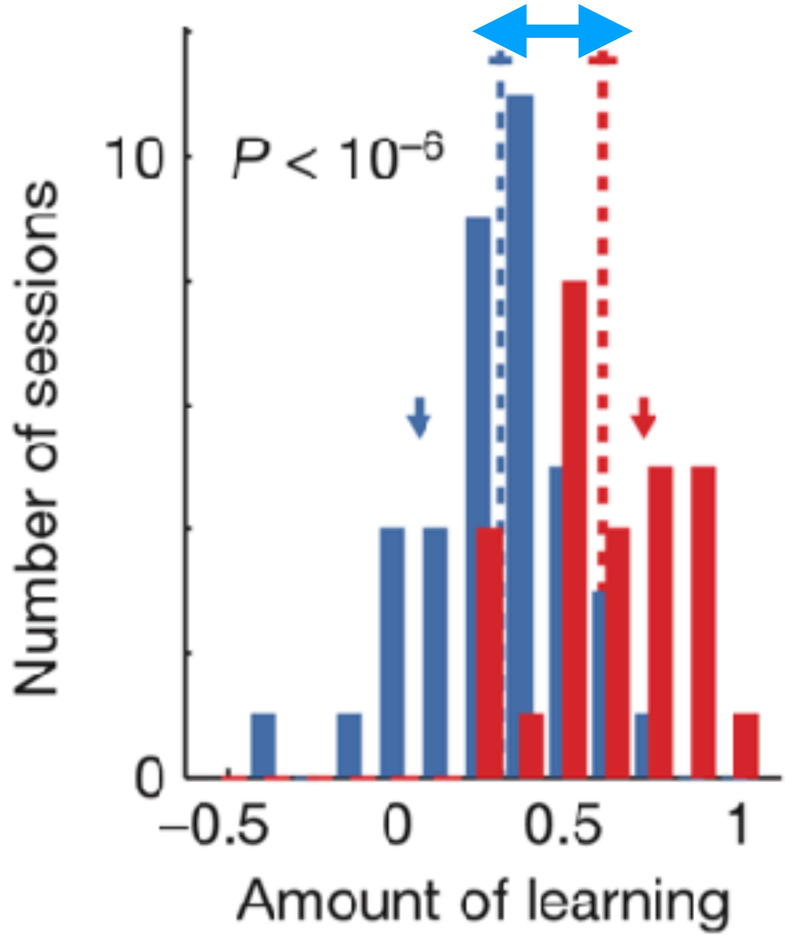
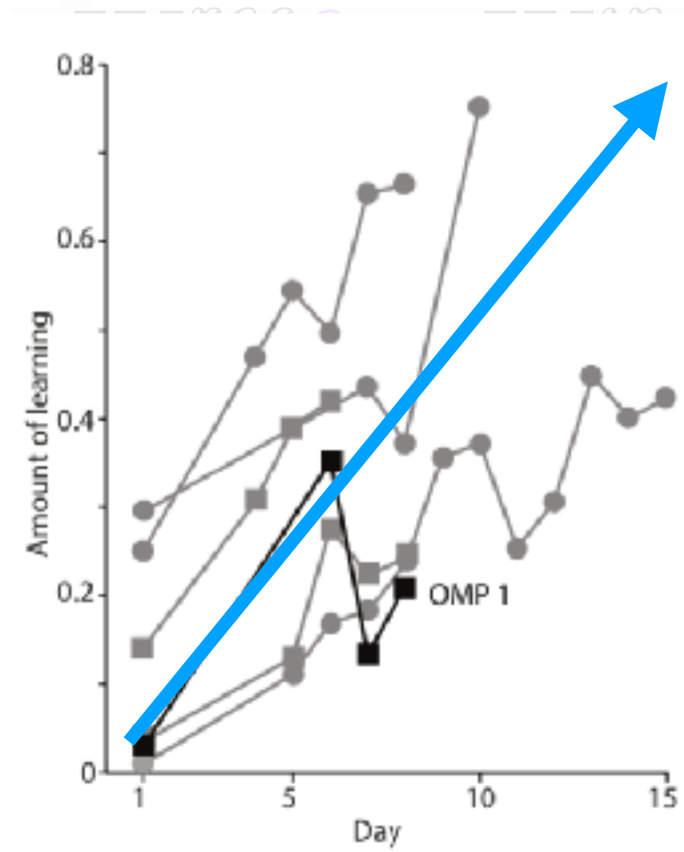
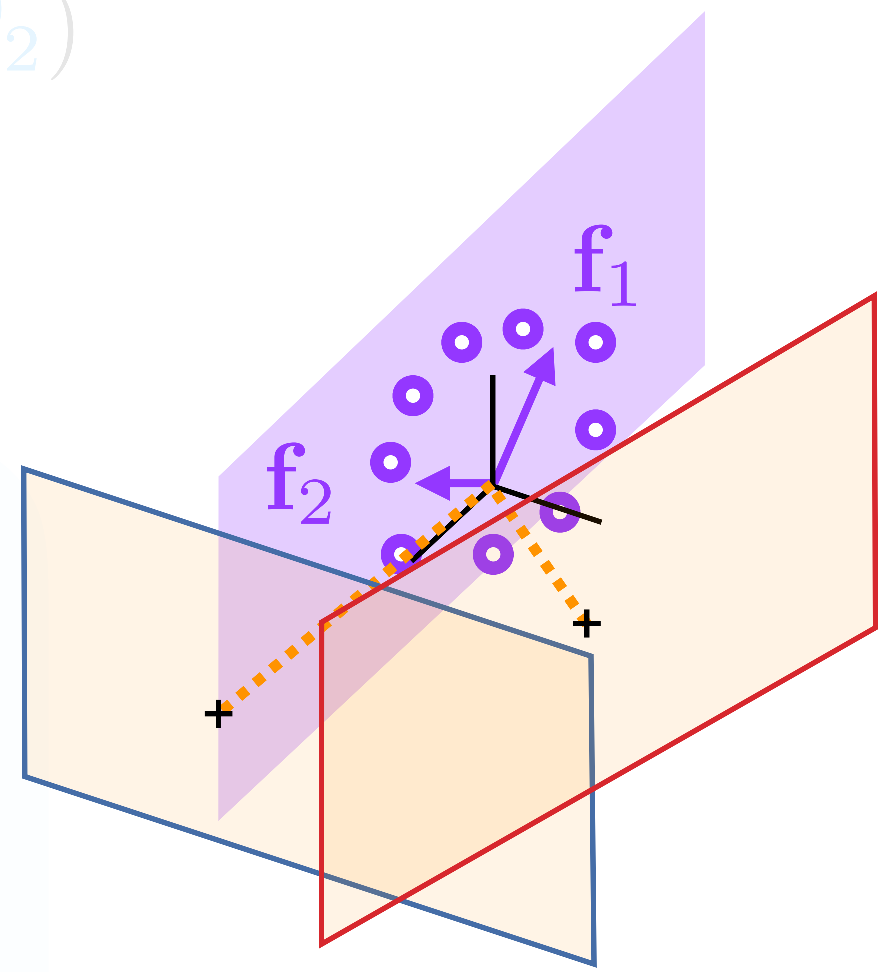
Sadtler et al. ’14



Oby et al. ’19





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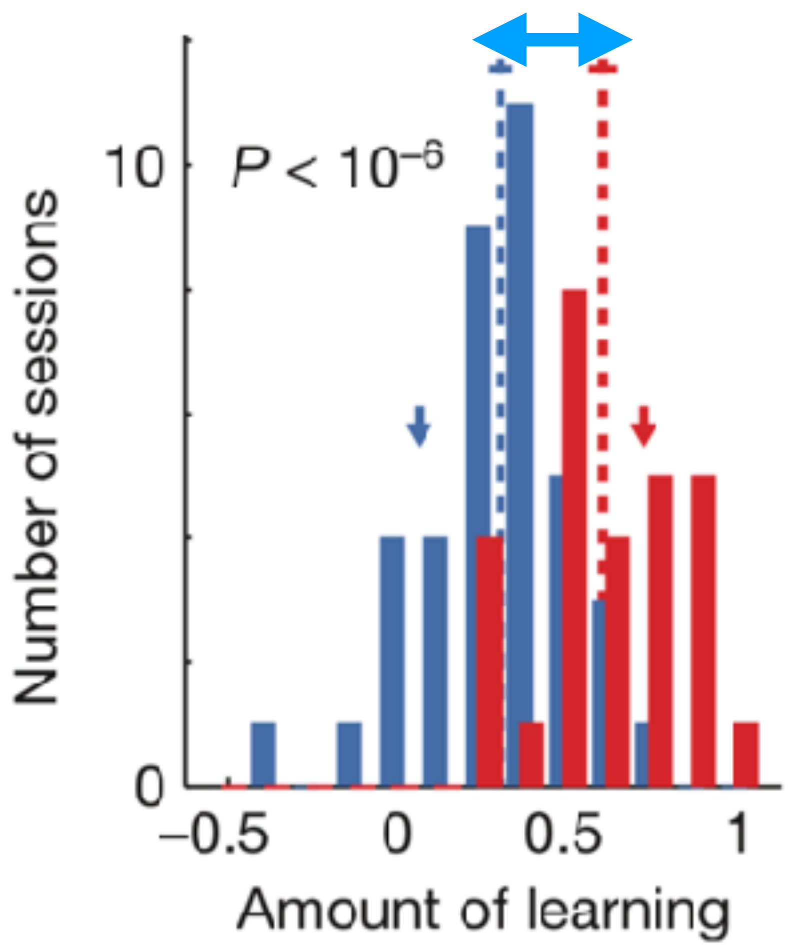
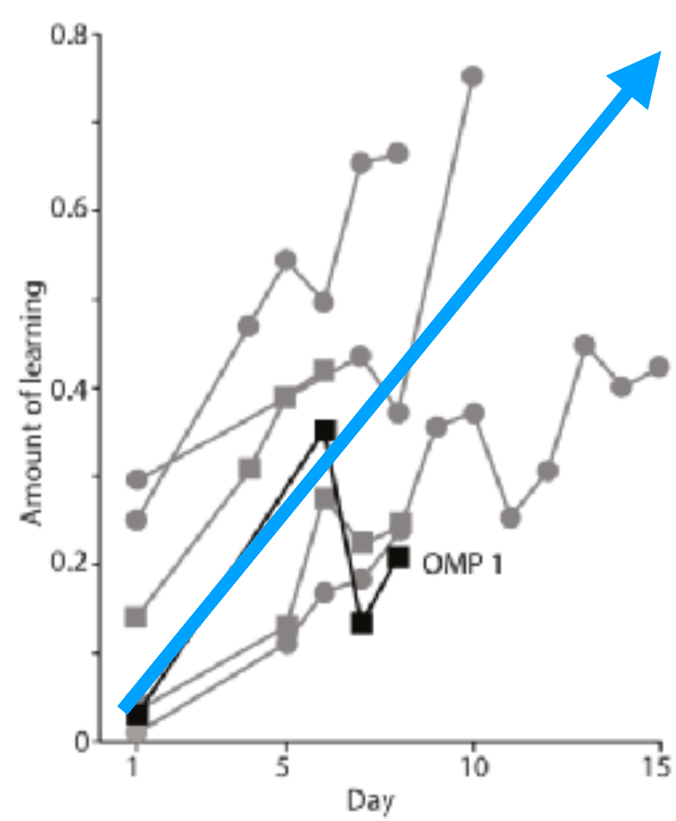
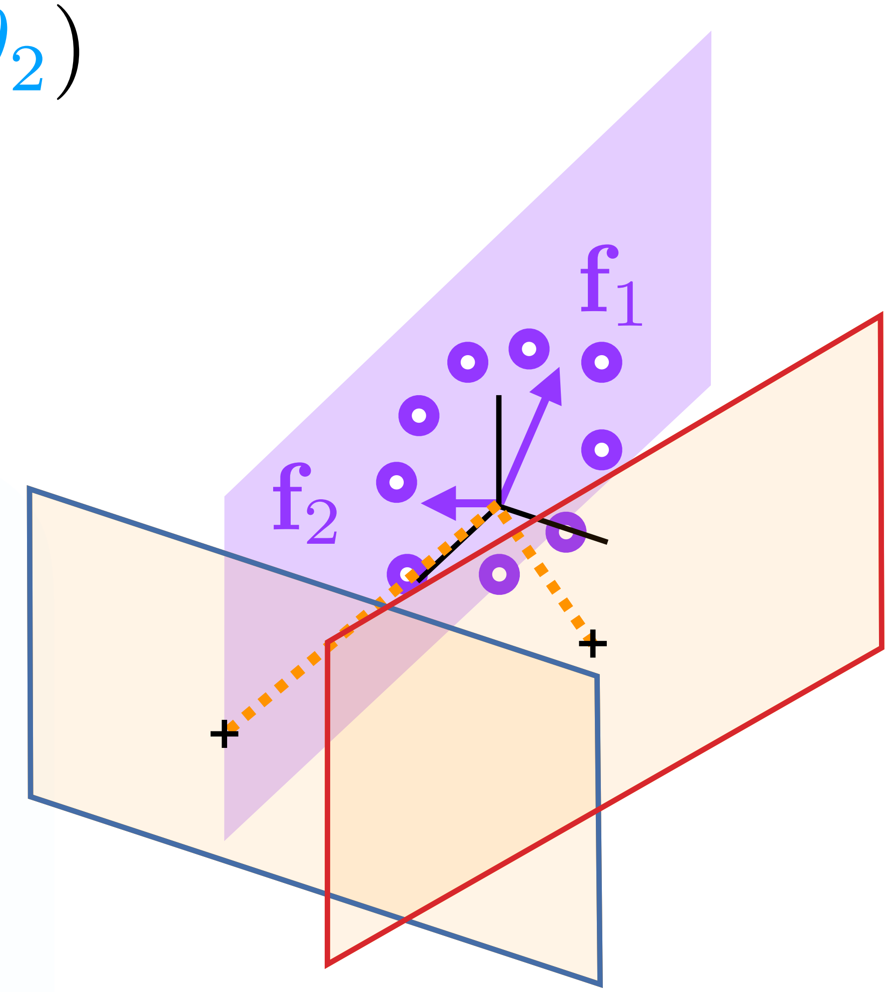


Oby et al. '19





(1) low-dimensional activity

(2) learning ~ alignment

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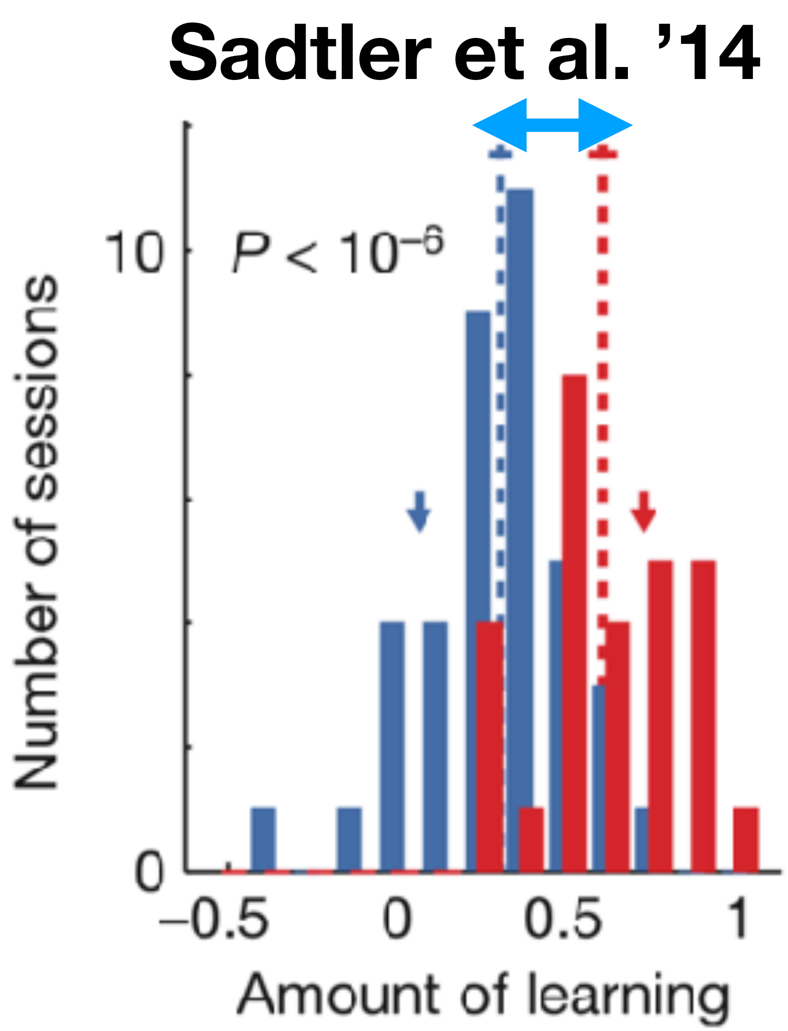
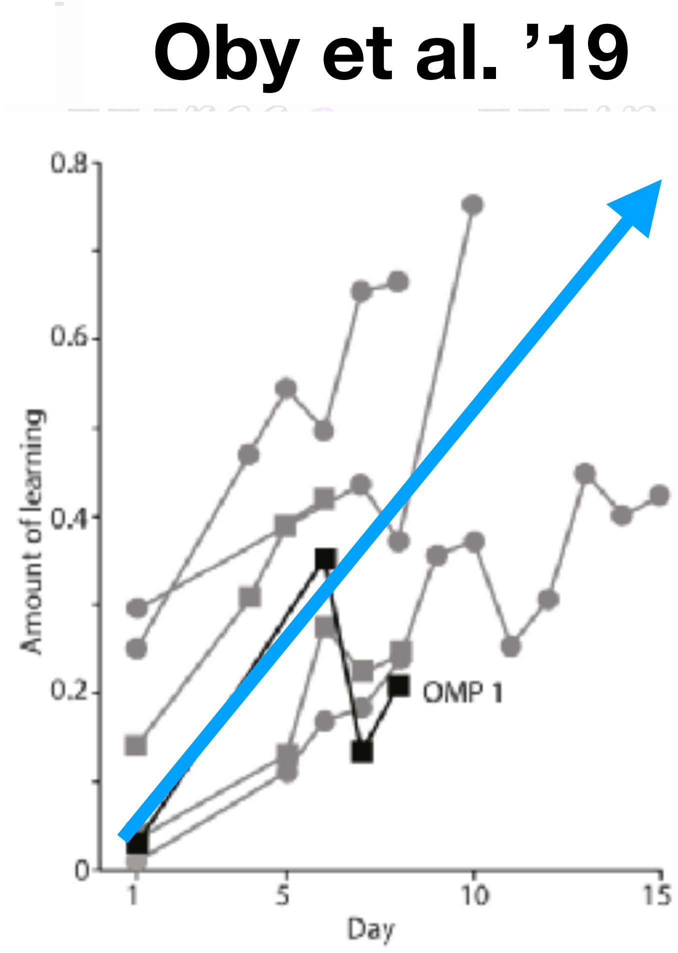
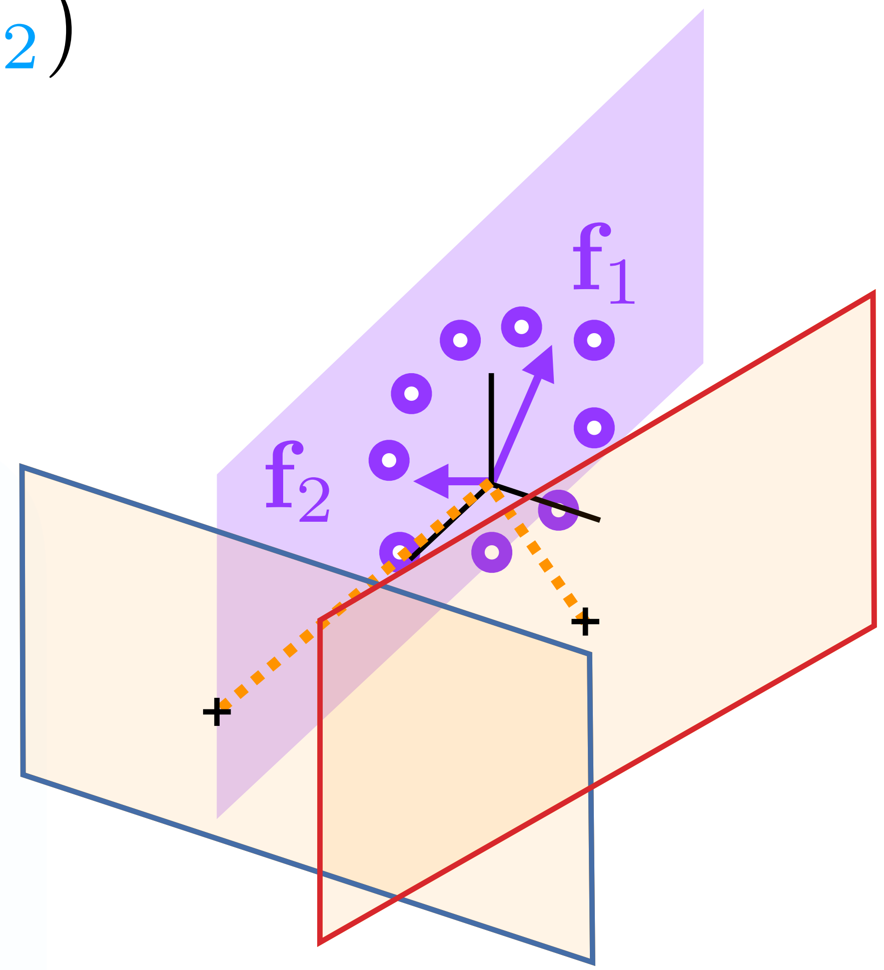
Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2 + \mathbf{m}_3\theta_3 +$

$$\tau\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}^{rec}\mathbf{x} + \mathbf{W}^{in}\left(\mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2\right)$$







(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2 + \mathbf{m}_3\theta_3 +$

$$\tau\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{W}^{rec}\mathbf{x} + \mathbf{W}^{in}\left(\mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2 + \mathbf{m}_3\theta_3 + \ldots\right)$$

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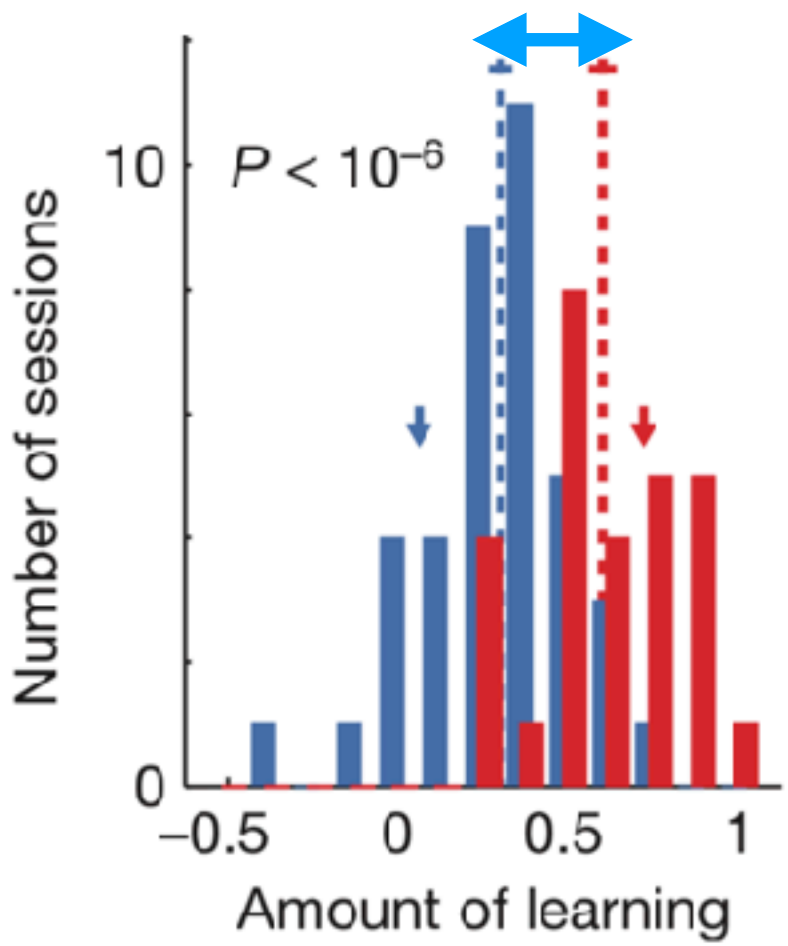
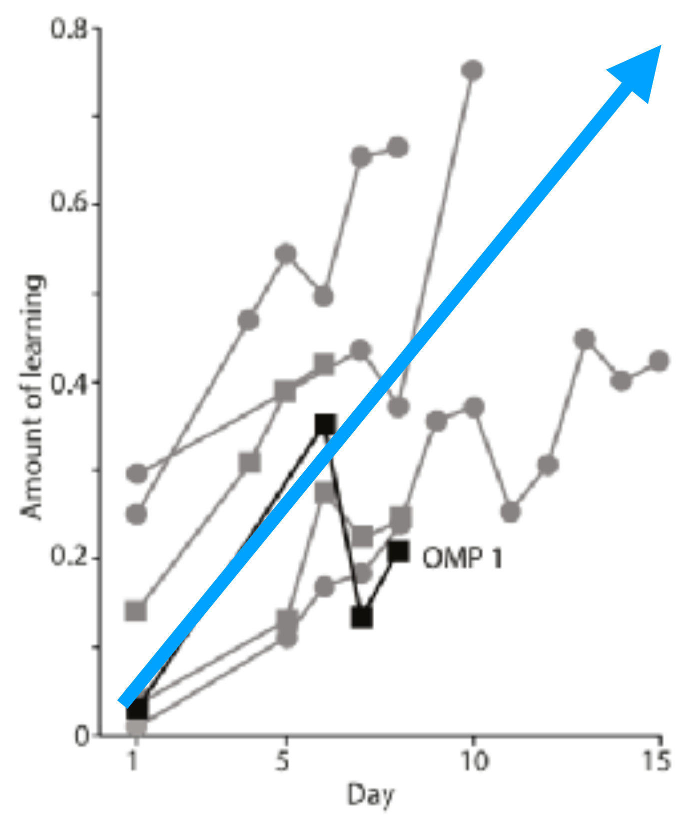
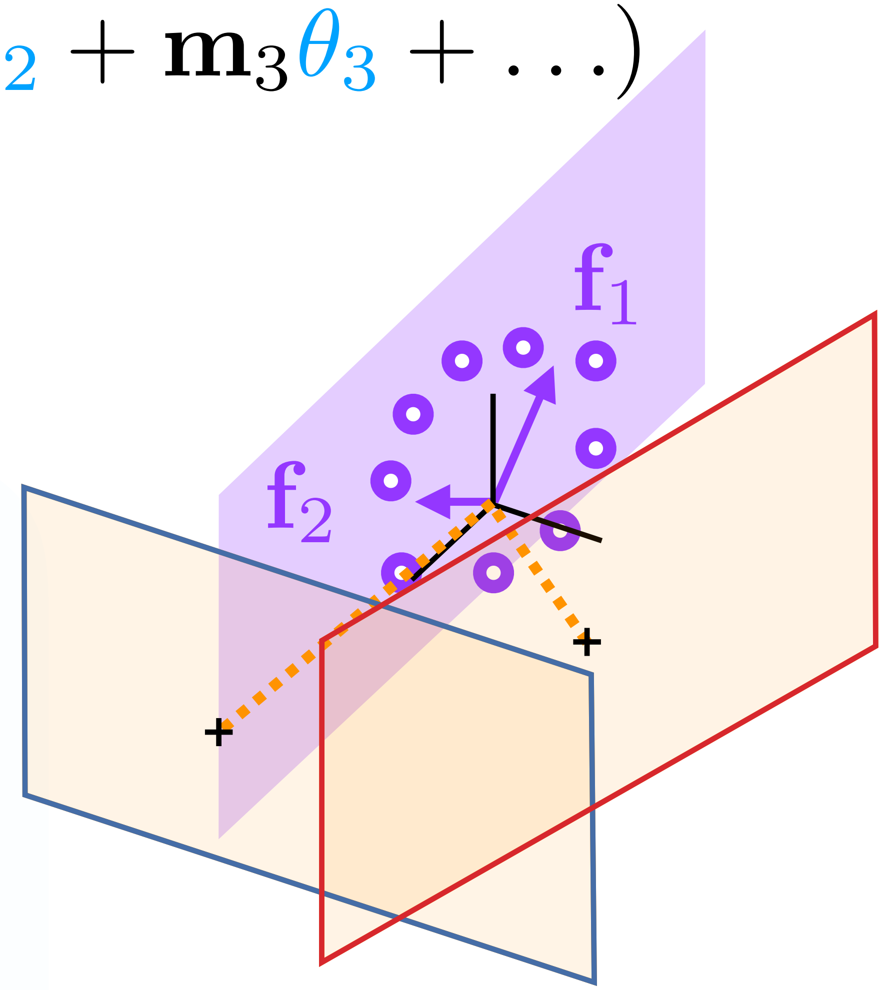


Oby et al. '19





(1) low-dimensional activity

(2) learning ~ alignment

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Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2 + \mathbf{m}_3\theta_3 +$

$$\Rightarrow \mathbf{x}_\theta(t) = \mathbf{f}_1\theta_1 + \mathbf{f}_2\theta_2 + \ldots + \mathbf{f}_K\theta_K$$

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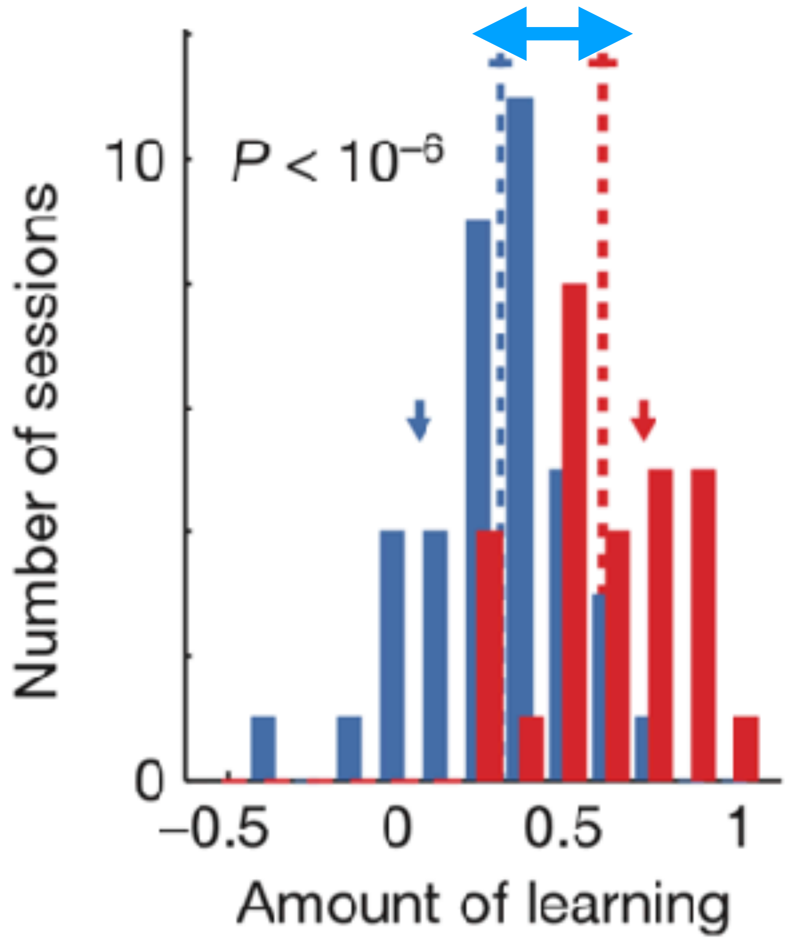
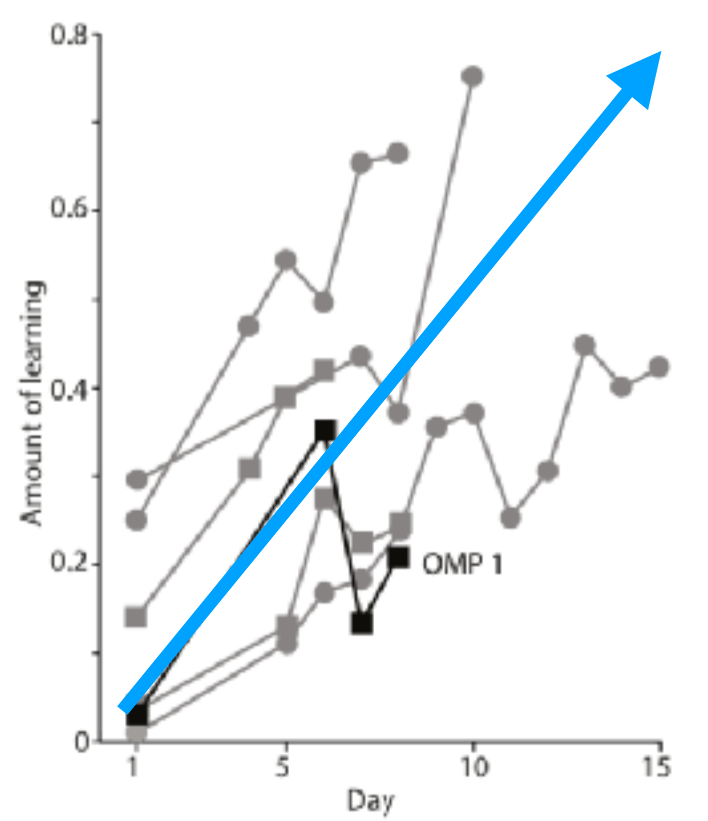
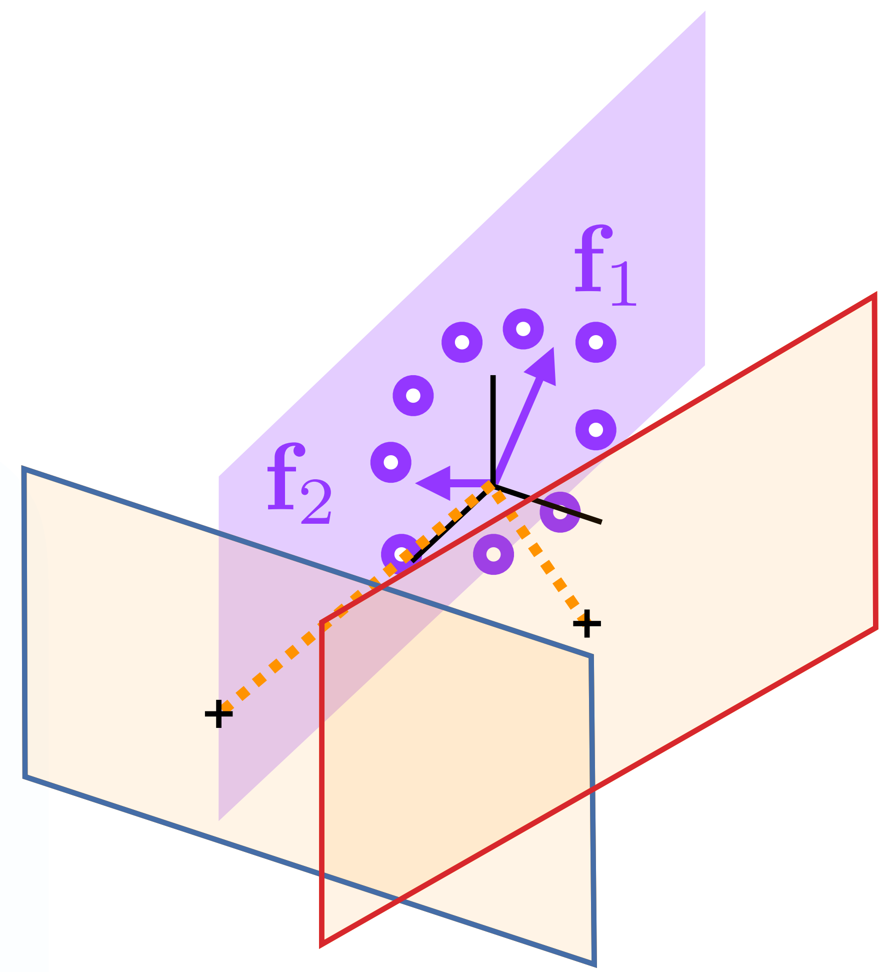


Oby et al. ’19





(1) low-dimensional activity

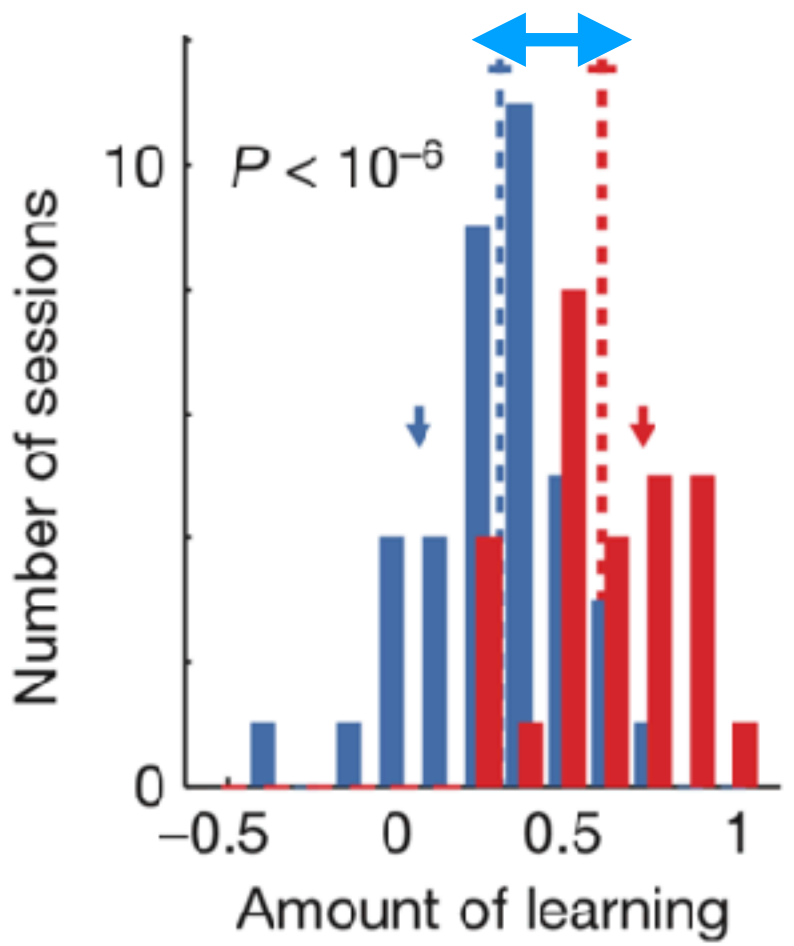
(2) learning ~ alignment

"re-aiming"

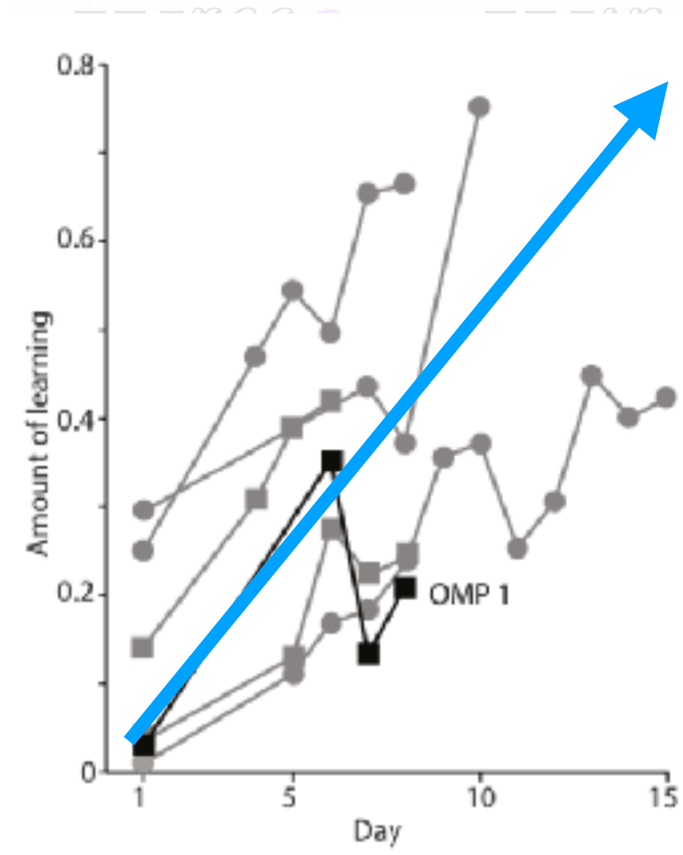
Linear dynamics: $\phi(\mathbf{x}) = \mathbf{x}, \quad \mathbf{u}_\theta = \mathbf{m}_1\theta_1 + \mathbf{m}_2\theta_2 + \mathbf{m}_3\theta_3 +$

$$\Rightarrow \mathbf{x}_\theta(t) = \mathbf{f}_1\theta_1 + \mathbf{f}_2\theta_2 + \ldots + \mathbf{f}_K\theta_K$$

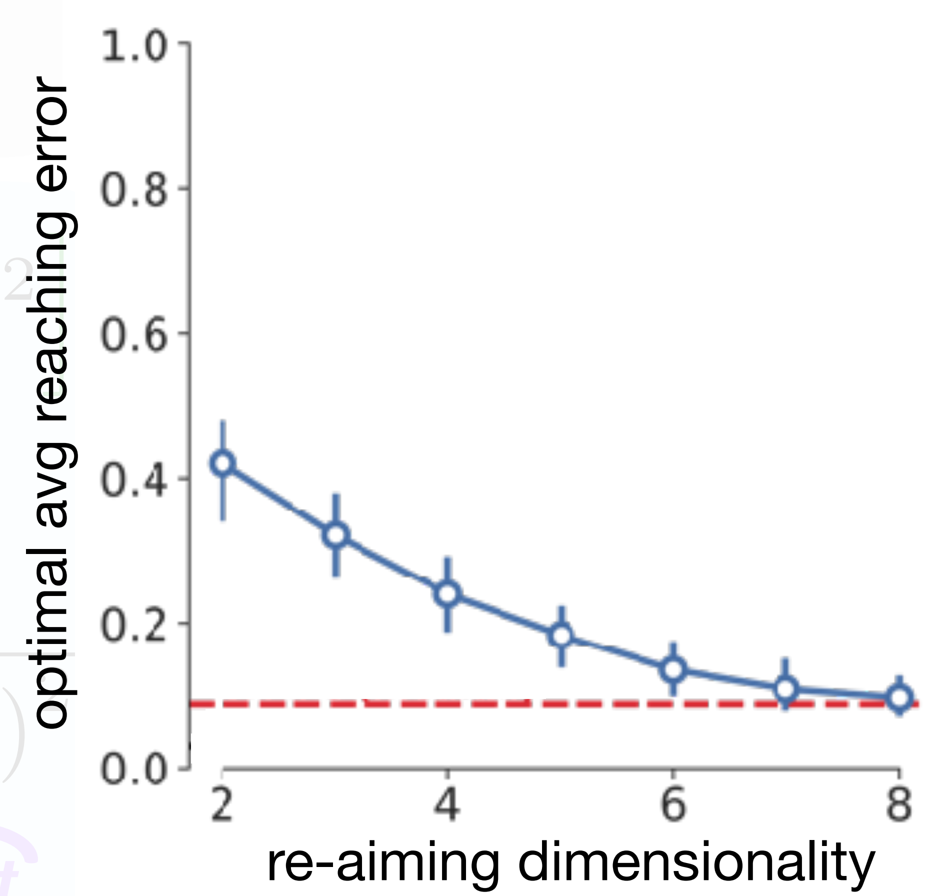
Sadtler et al. '14



Oby et al. '19



Menendez et al. '20



(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

Non-linear dynamics?

(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

Non-linear dynamics?

$$\phi(\mathbf{x}) = \mathrm{ReLU}(\mathbf{x}), \quad \mathbf{u}_{\theta} = \phi\left(\mathbf{M}\boldsymbol{\theta}\right)$$

(1) low-dimensional activity

(2) learning ~ alignment

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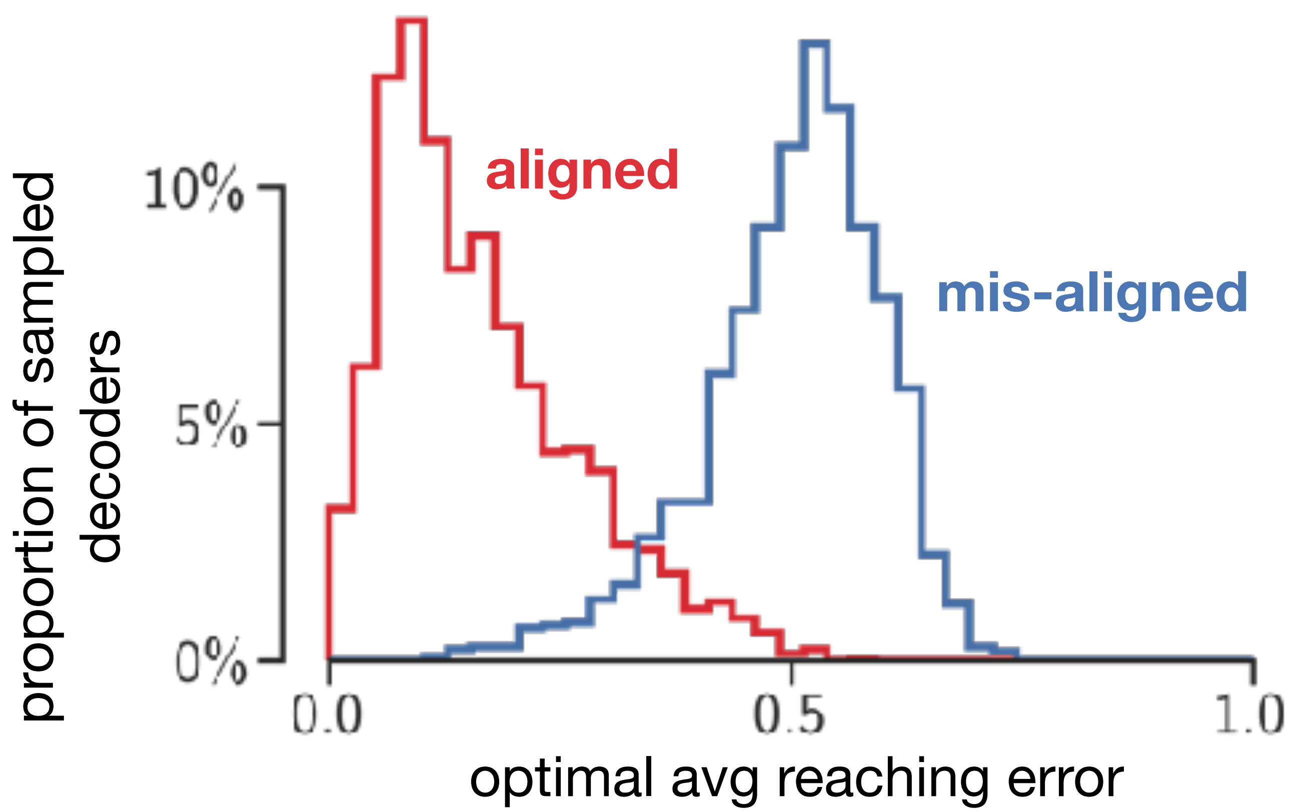

(1) low-dimensional activity

(2) learning ~ alignment

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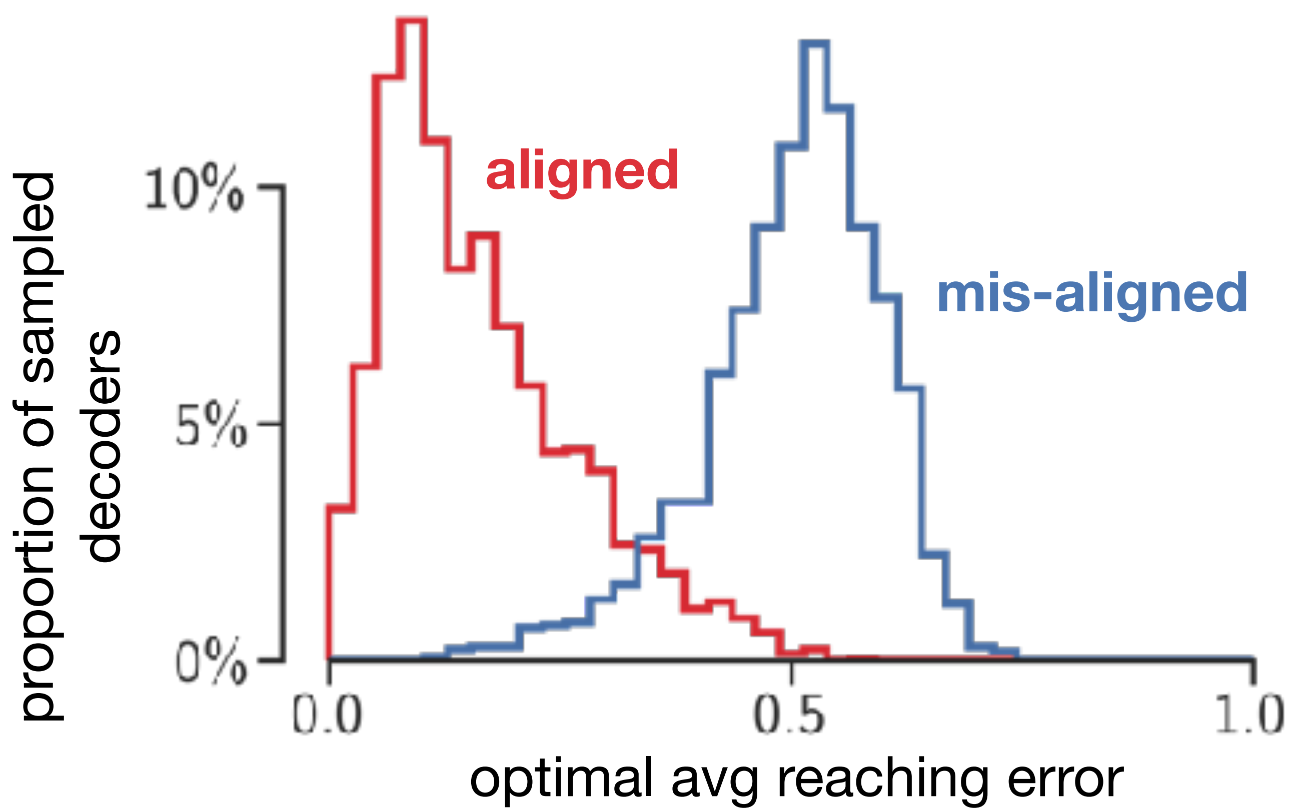

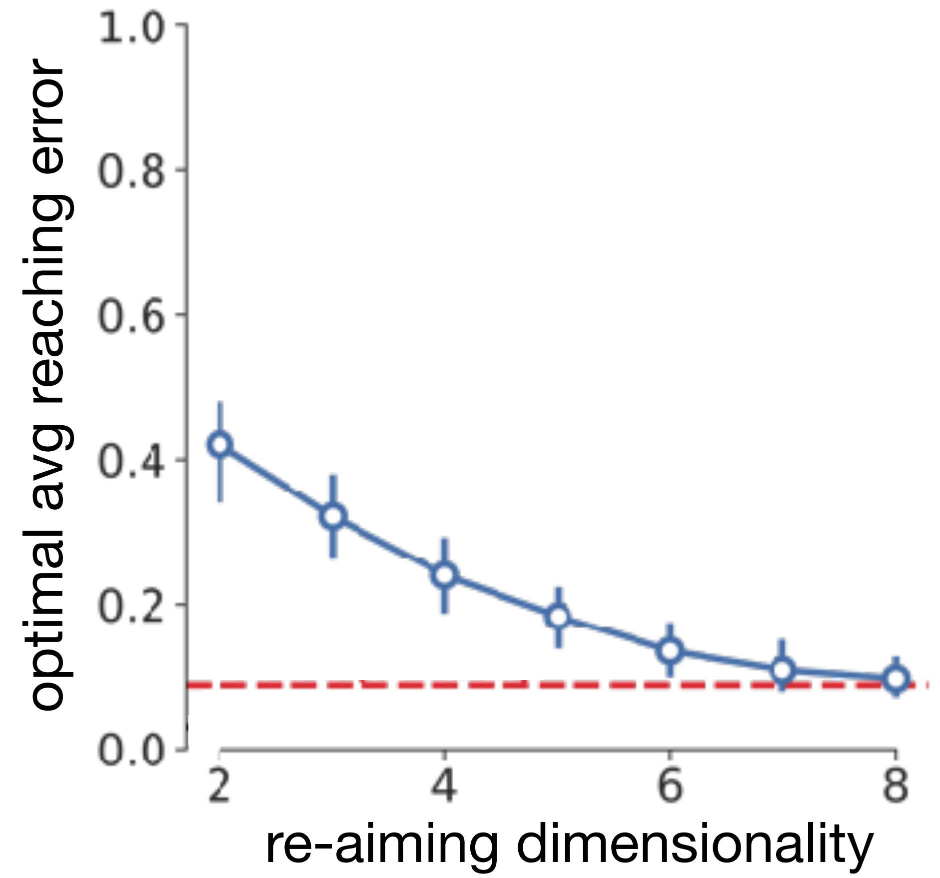

(1) low-dimensional activity

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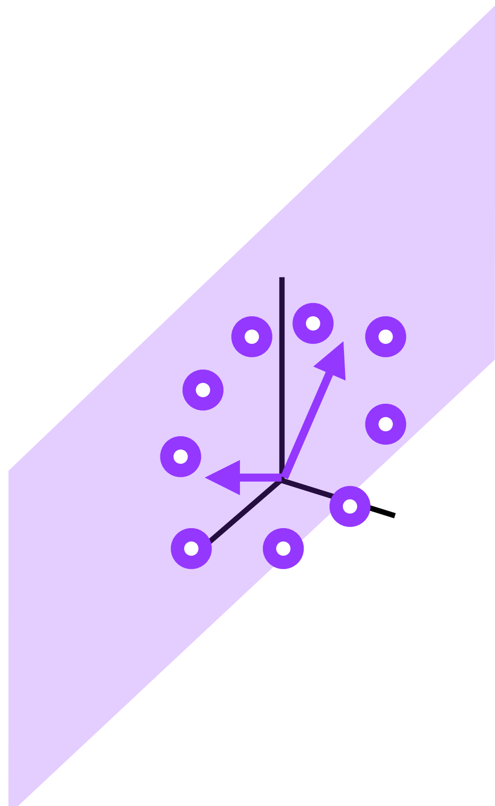

(1) low-dimensional activity

(2) learning ~ alignment

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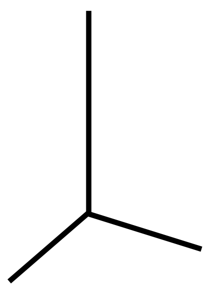


(1) low-dimensional activity

(2) learning ~ alignment

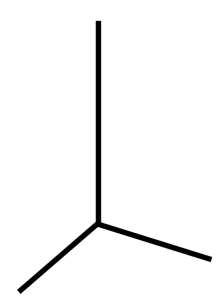
}

"re-aiming"

Non-linear dynamics?

$$\phi(\mathbf{x}) = \mathrm{ReLU}(\mathbf{x}), \quad \mathbf{u}_{\theta} = \phi\left(\mathbf{M}\boldsymbol{\theta}\right)$$

2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$



(1) low-dimensional activity

(2) learning ~ alignment

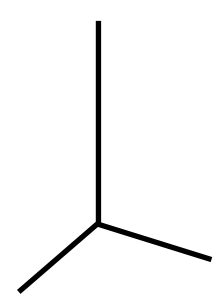
"re-aiming"

Non-linear dynamics?

$$\phi(\mathbf{x}) = \mathrm{ReLU}(\mathbf{x}), \quad \mathbf{u}_{\theta} = \phi\left(\mathbf{M}\boldsymbol{\theta}\right)$$

2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle



(1) low-dimensional activity

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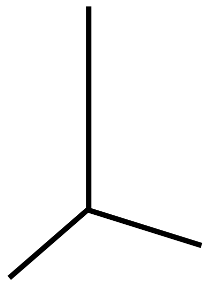
Non-linear dynamics?

$$\phi(\mathbf{x}) = \mathrm{ReLU}(\mathbf{x}), \quad \mathbf{u}_{\theta} = \phi\left(\mathbf{M}\boldsymbol{\theta}\right)$$

2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



(1) low-dimensional activity

(2) learning ~ alignment

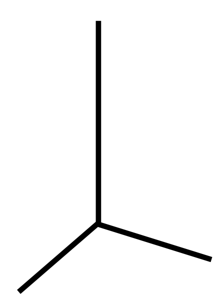
"re-aiming"

Non-linear dynamics?

$$\phi(\mathbf{x}) = \mathrm{ReLU}(\mathbf{x}), \quad \mathbf{u}_{\theta} = \phi\left(\mathbf{M}\boldsymbol{\theta}\right)$$

2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



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2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

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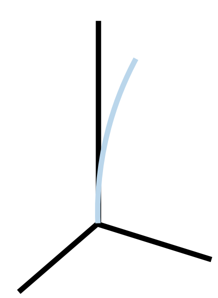

(1) low-dimensional activity

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2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

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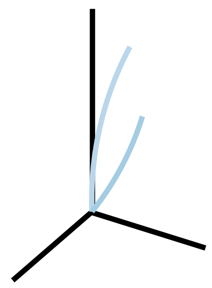

(1) low-dimensional activity

(2) learning ~ alignment

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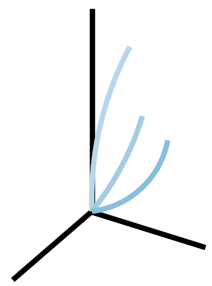
2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



(1) low-dimensional activity

(2) learning ~ alignment

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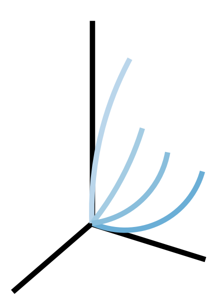
- magnitude



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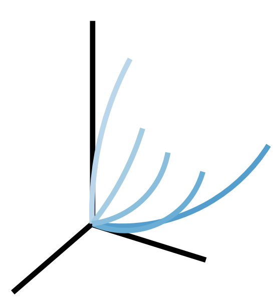

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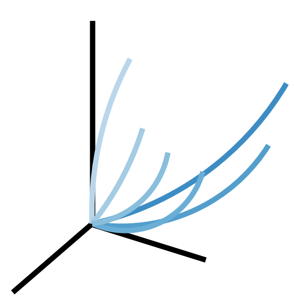
2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



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2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

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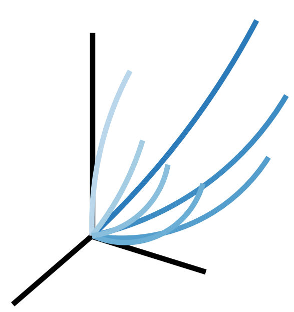

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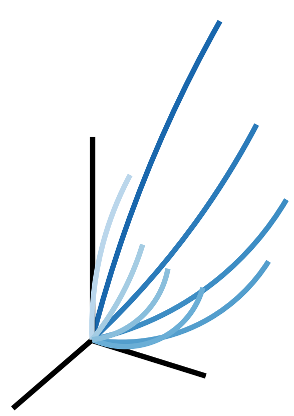

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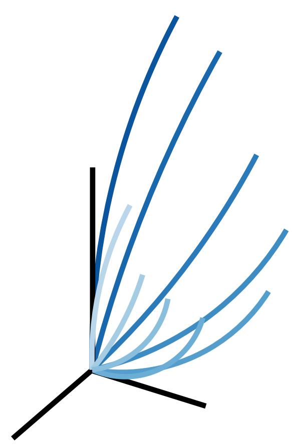

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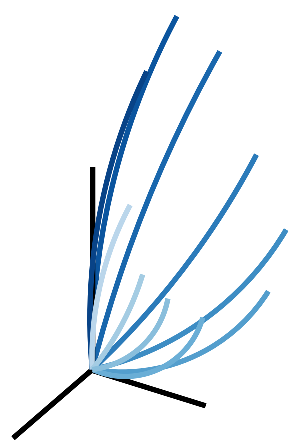

(1) low-dimensional activity

(2) learning ~ alignment

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Non-linear dynamics?

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- angle
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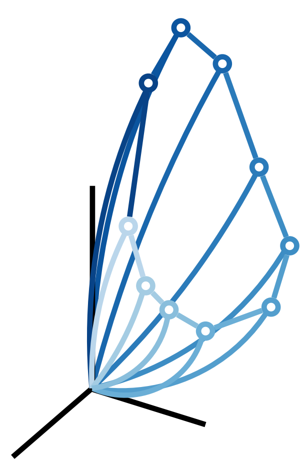

(1) low-dimensional activity

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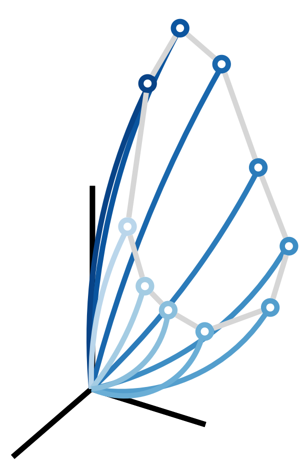

(1) low-dimensional activity

(2) learning ~ alignment

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Non-linear dynamics?

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2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

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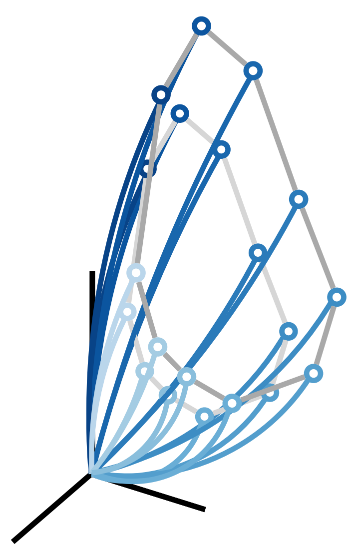
- magnitude



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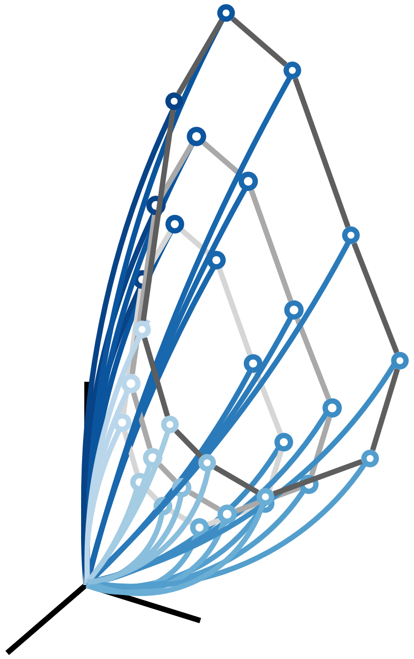
- magnitude



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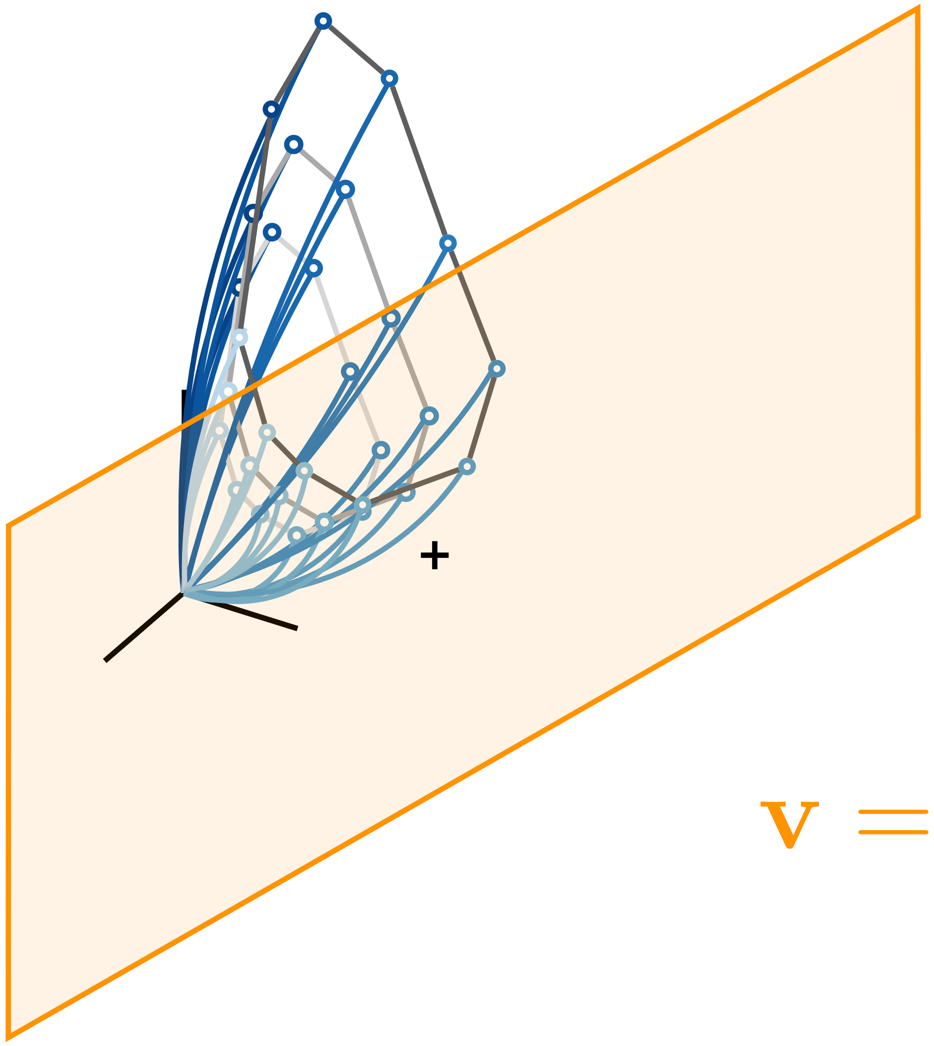
- angle
- magnitude



$$\mathbf{v} = \mathbf{D}\phi(\mathbf{x})$$

(1) low-dimensional activity

(2) learning ~ alignment

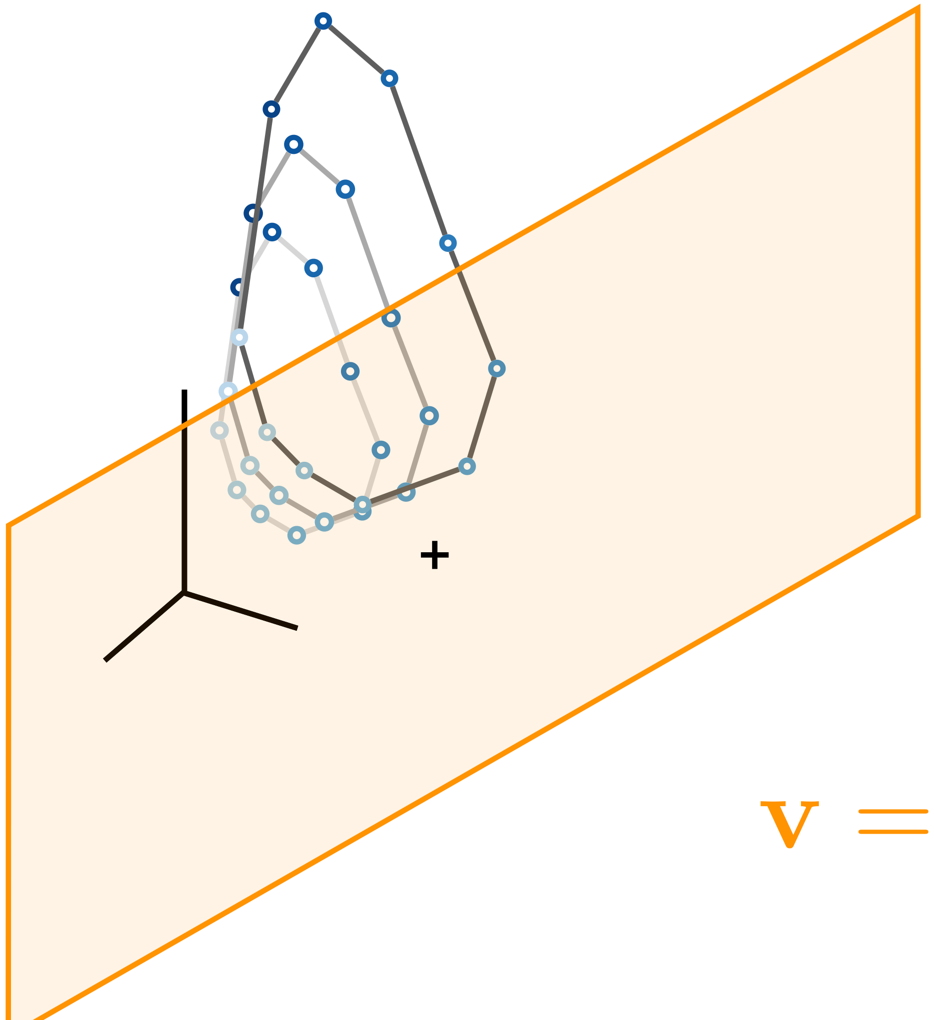
"re-aiming"

Non-linear dynamics?

$$\phi(\mathbf{x}) = \mathrm{ReLU}(\mathbf{x}), \quad \mathbf{u}_\theta = \phi\left(\mathbf{M}\boldsymbol{\theta}\right)$$

2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



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(2) learning ~ alignment

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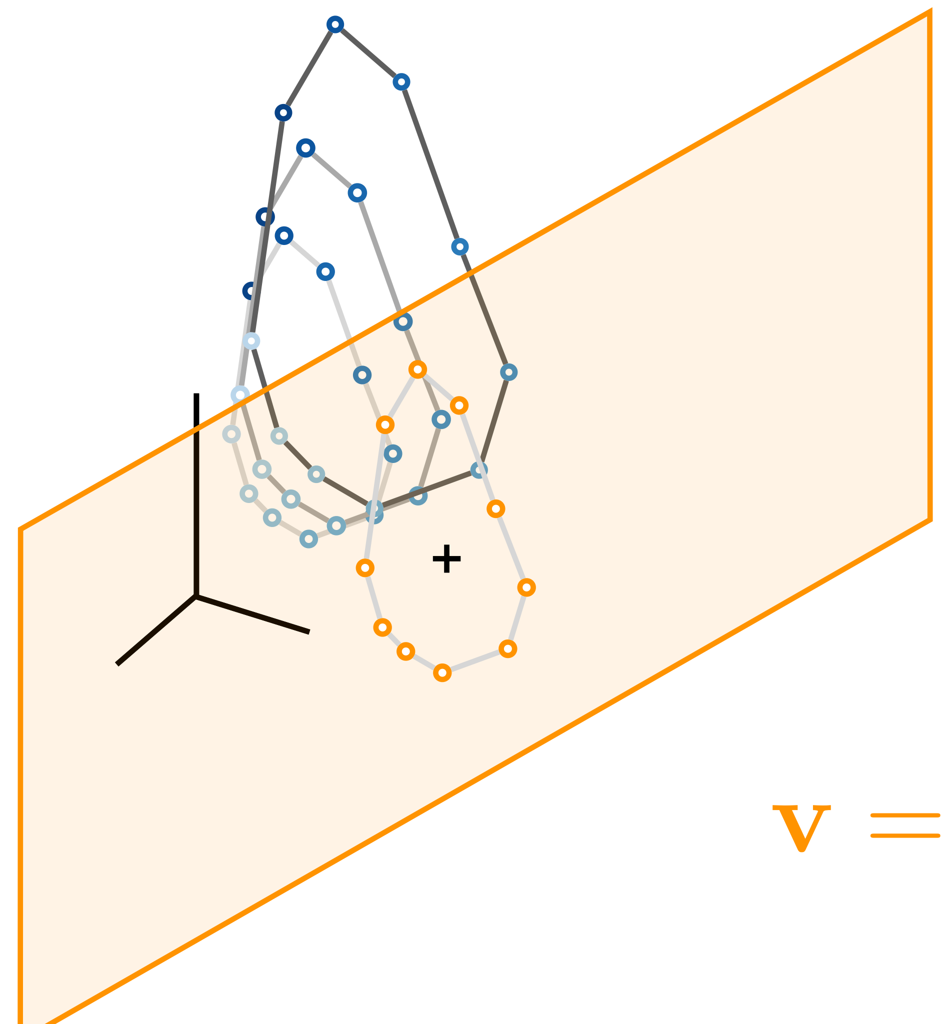
2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



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(1) low-dimensional activity

(2) learning ~ alignment

} **"re-aiming"**

Non-linear dynamics?

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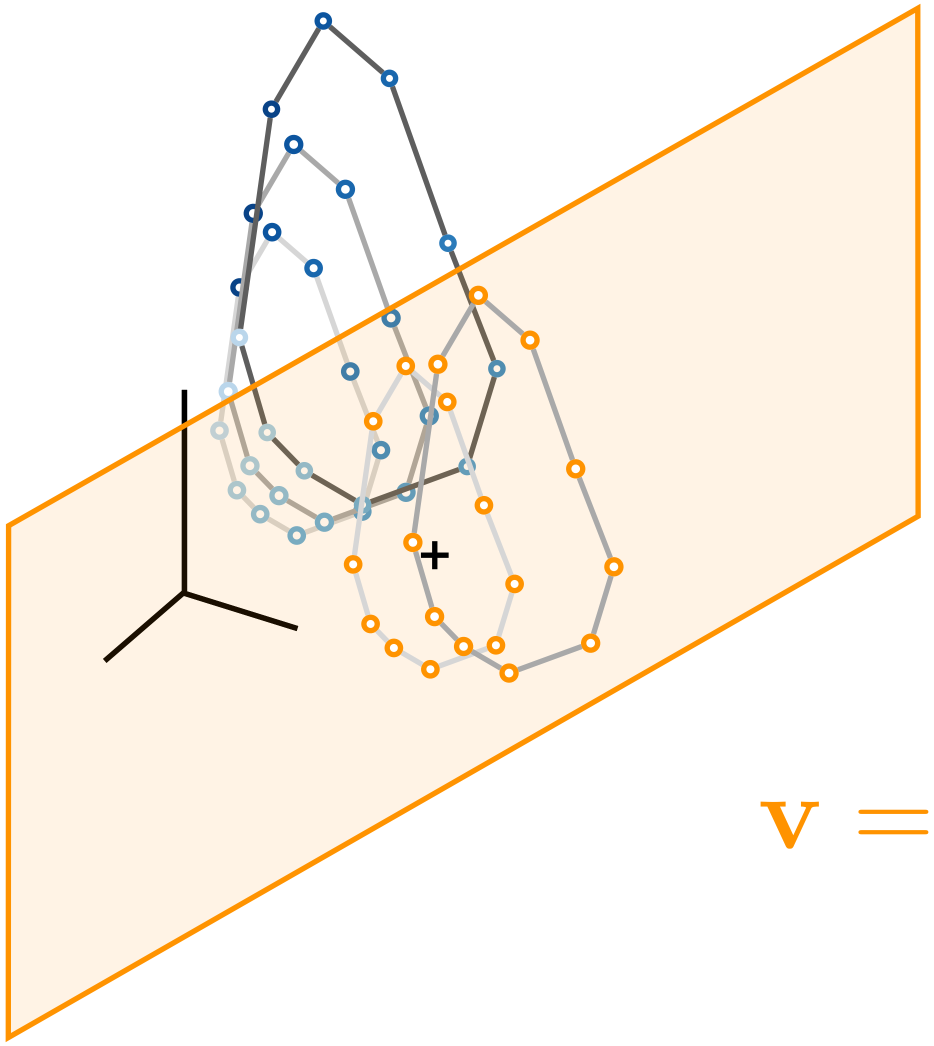
2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



$\mathbf{v} = \mathbf{D}\phi(\mathbf{x})$

(1) low-dimensional activity

(2) learning ~ alignment

"re-aiming"

Non-linear dynamics?

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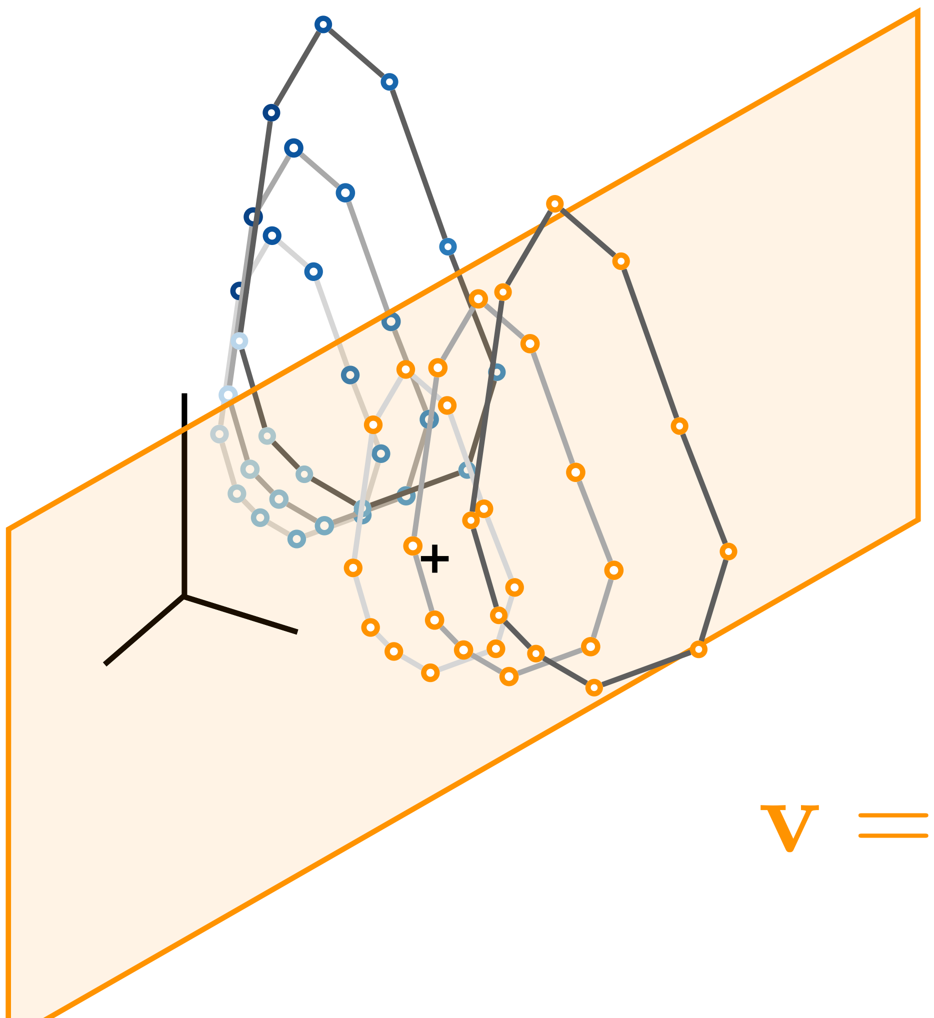
2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



$$\mathbf{v} = \mathbf{D}\phi(\mathbf{x})$$

(1) low-dimensional activity

(2) learning ~ alignment

(3) behavioral asymmetry

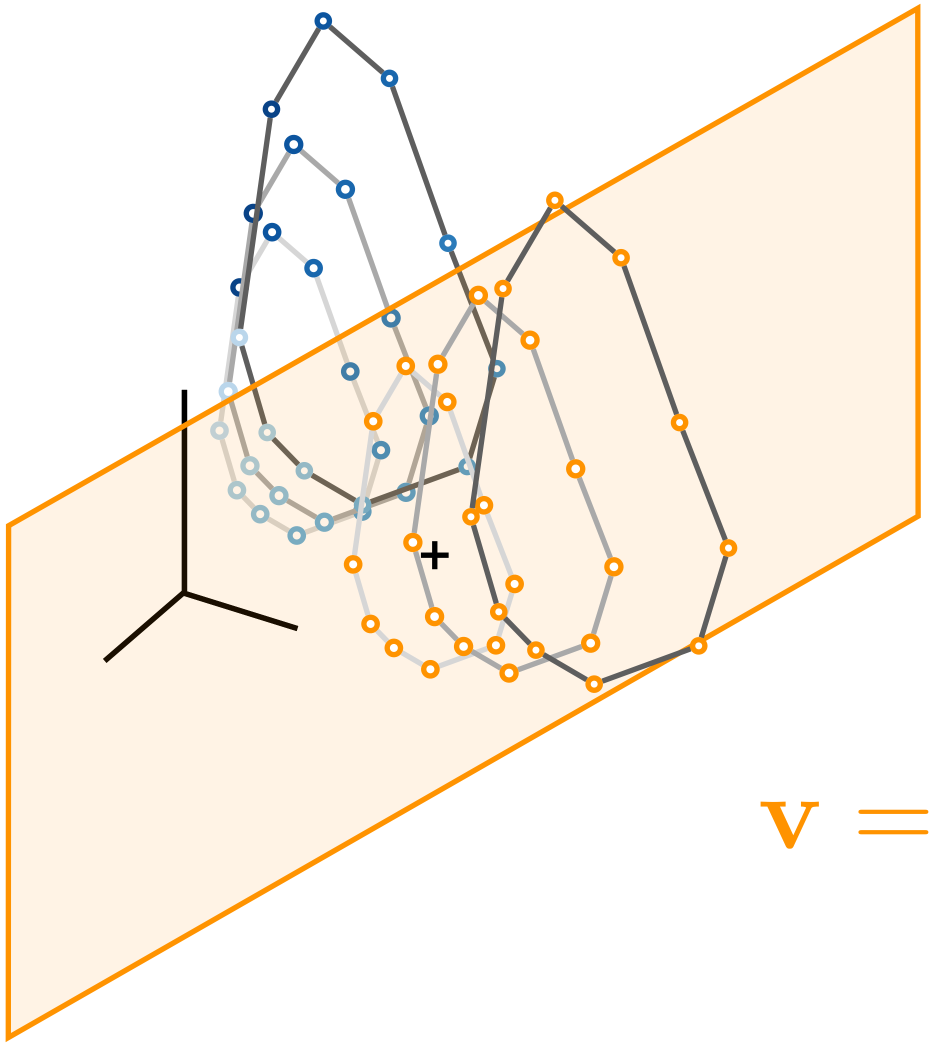
"re-aiming"

Non-linear dynamics?

$$\phi(\mathbf{x}) = \mathrm{ReLU}(\mathbf{x}), \quad \mathbf{u}_{\theta} = \phi\left(\mathbf{M}\boldsymbol{\theta}\right)$$

2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude



$$\mathbf{v} = \mathbf{D}\phi(\mathbf{x})$$

(1) low-dimensional activity
(2) learning ~ alignment
(3) behavioral asymmetry

"re-aiming"

Non-linear dynamics?

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2D re-aiming: $\boldsymbol{\theta} \in \mathbb{R}^2$

- angle
- magnitude

$$\mathbf{v} = \mathbf{D}\phi(\mathbf{x})$$

(1) low-dimensional activity

(2) learning ~ alignment

(3) behavioral asymmetry

"re-aiming"

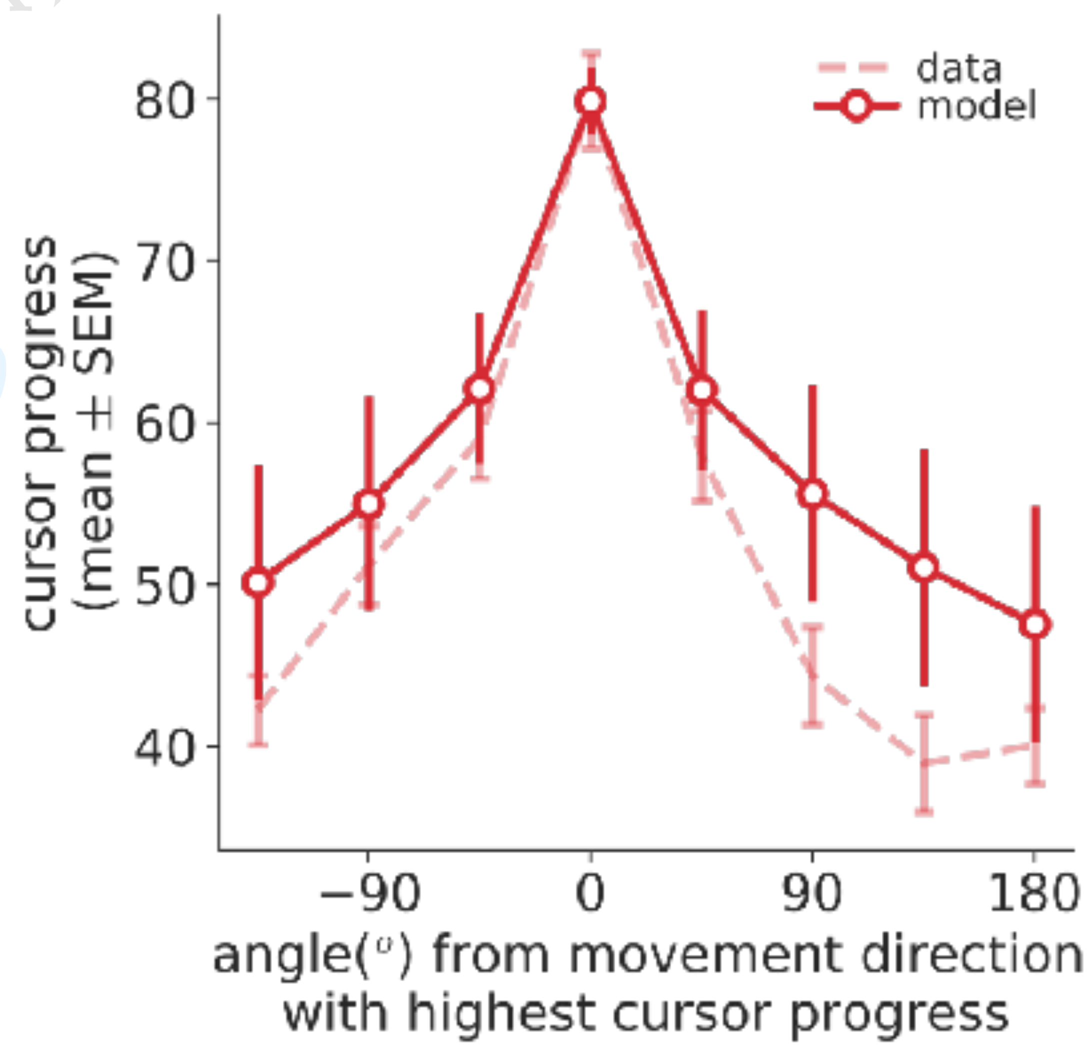
Non-linear dynamics?

2D re-aiming: θ

- angle
- magnitude

$\mathbf{v} = \mathbf{D}\phi(\mathbf{x})$

Menendez, Hennig, Golub, Oby, Batista, Chase, Yu & Latham (*Cosyne*, 2020)



(1) low-dimensional activity

(2) learning ~ alignment

(3) behavioral asymmetry

} **“re-aiming”**

Future directions

(1) low-dimensional activity

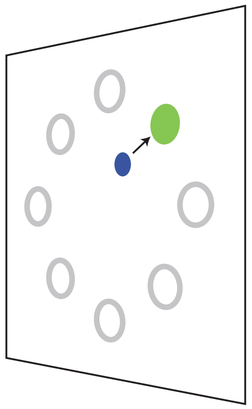
(2) learning ~ alignment

(3) behavioral asymmetry

"re-aiming"

Future directions

closed-loop control



(1) low-dimensional activity

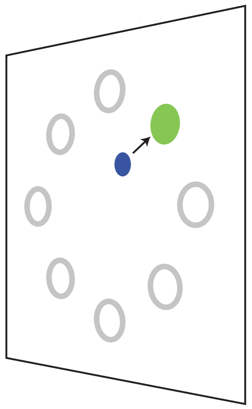
(2) learning ~ alignment

(3) behavioral asymmetry

“re-aiming”

Future directions

closed-loop control



$$\boldsymbol{\theta}^*(t) = \underset{\boldsymbol{\theta}(t)}{\arg\min} \int_t^T \|\mathbf{D}\mathbf{x}(\tau) - \mathbf{v}^*\|^2 \mathrm{d}\tau$$

(1) low-dimensional activity

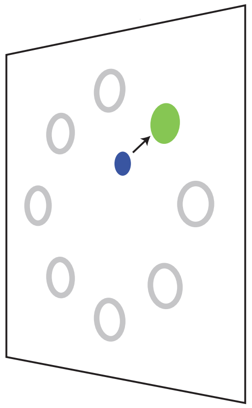
(2) learning ~ alignment

(3) behavioral asymmetry

} **“re-aiming”**

Future directions

closed-loop control



$$\boldsymbol{\theta}^*(t) = \arg\min_{\boldsymbol{\theta}(t)} \int_t^T \|\mathbf{D}\mathbf{x}(\tau) - \mathbf{v}^*\|^2 \mathrm{d}\tau$$

what is learned vs. ***how*** it is learned

(1) low-dimensional activity

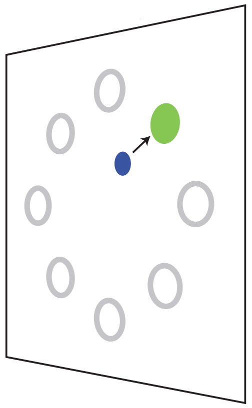
(2) learning ~ alignment

(3) behavioral asymmetry

"re-aiming"

Future directions

closed-loop control



$$\boldsymbol{\theta}^*(t) = \arg\min_{\boldsymbol{\theta}(t)} \int_t^T \|\mathbf{D}\mathbf{x}(\tau) - \mathbf{v}^*\|^2 \mathrm{d}\tau$$

what is learned vs. ***how*** it is learned

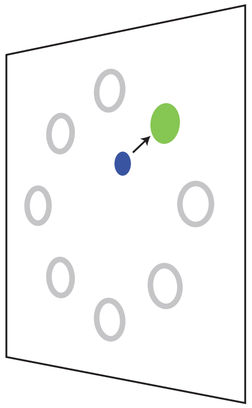
what does this tell us about **motor learning**?

(1) low-dimensional activity
(2) learning ~ alignment
(3) behavioral asymmetry

} **"re-aiming"**

Future directions

closed-loop control



$$\theta^*(t) = \underset{\theta(t)}{\arg\min} \int_t^T \|\mathbf{D}\mathbf{x}(\tau) - \mathbf{v}^*\|^2 d\tau$$

what is learned vs. ***how*** it is learned

what does this tell us about **motor learning**?

re-aiming = learning a **sensorimotor map**

(1) low-dimensional activity

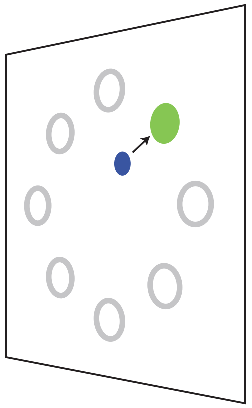
(2) learning ~ alignment

(3) behavioral asymmetry

} **"re-aiming"**

Future directions

closed-loop control



$$\boldsymbol{\theta}^*(t) = \arg\min_{\boldsymbol{\theta}(t)} \int_t^T \|\mathbf{D}\mathbf{x}(\tau) - \mathbf{v}^*\|^2 \mathrm{d}\tau$$

what is learned vs. ***how*** it is learned

what does this tell us about **motor learning**?

re-aiming = learning a **sensorimotor map**

low-dimensional **learning strategies**

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Thank you!